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# Analysis of Mathematics Student Teachers' Conceptions of the Limit of a Function: An Autoethnographic Perspective

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**ABSTRACT**

The initial teacher education is the focal point for developing student teachers' content knowledge and the pedagogy that goes with it. The limit concept, amongst other concepts, allowed us to model what it means to teach mathematical concepts for understanding. The concept of a limit plays a significant role in the study of calculus. Although there is extensive research that focuses on students' difficulties related to the limit of a function, the literature is scarce on students' conceptions of this important topic. As a result, the study analyses the mathematics student teachers' conceptions of the limit of a function. An autoethnographic research design was to foreground the conceptions of three student teachers through a limit test based on the intuitive definition of a limit, its application using a table of values, its graphical determination, and its algebraic determination. The thematic analysis procedure was used to analyze student teachers' test scripts. The findings of the study revealed that student teachers have underdeveloped conceptions of the intuitive definition of the limit concept. It was also revealed that some possess conceptions of the algebraic representations, as compared to other forms of representations. It is concluded that mathematics student teachers should develop different representations of the limit concept to create meaningful learning opportunities for their prospective learners.

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## INTRODUCTION

Initial Teacher Education (ITE) is a focal point for developing student teachers' pedagogical content knowledge of the limit concept and other mathematics concepts. In our view, student teachers' nuanced understanding of the limit concept during ITE is crucial for two reasons. First, their developed conceptions of the concept serve as a strong foundation for their own success in subsequent calculus topics. Student teachers who have a nuanced understanding of the limit concept can easily solve limit-based calculus problems. Secondly, student teachers need to demonstrate a high level of content mastery and be able to teach in a meaningful way to their prospective mathematics learners. The contention is that student teachers need to move beyond a procedural understanding of the limit concept by engaging with multiple representations of the concept (intuitive, numerical, graphical, and algebraic) to develop their conceptions. At a broader level, the engagement with the limit concept provided us with an opportunity to take the student teachers through what it means to teach a mathematical concept for understanding.

The concept of a limit plays a significant role in the study of calculus. Salas et al. (2021) argued that "without limit, calculus does not exist; every single notion of calculus is a limit in one sense or another" (p. 1). Hence, it is not surprising that the limit concept is often considered as a foundation for continuity, differentiation, and integration (Bansilal & Mkhwanazi, 2022; Munyaruhengeri et al., 2025) and mathematical analysis (Ableitinger et al., 2023; Chand et al., 2024; Trapezanlidis & Nikolantonakis, 2026). Nonetheless, students' difficulties with the limit concept have been widely documented (Baye et al., 2021; Arsyad et al., 2021; Jameson et al., 2023; Nurdin et al., 2021; Sebsibe, 2019). Baye et al. (2021) highlighted students' lack of conceptual understanding of this key concept in mathematics. Likewise, previous studies conducted revealed that students characterize limits as unreachable, approximate, and as a boundary (Sebsibe & Feza, 2019; Sulastri et al., 2021; Trapezanlidis & Nikolantonakis, 2026; Usman et al., 2025). Such characterization suggests the "influence of everyday meanings of the word 'limit' on students' conceptions" (Bansilal & Mkhwanazi, 2022, p. 2084). Furthermore, students perceive the limit and the function values as the same (Adhikari, 2020; Messias, 2024). This conceptual challenge was also noted in an earlier study conducted by Denbel (2014), which highlighted that "students think that when a function has a limit, then it has to be continuous at that point" (p. 24). Amongst the plausible explanations for these challenges is that the limit concept is an abstract and complex notion (Baye et al., 2021; Díaz, 2022), suggesting that defining and understanding a limit is not easy for students. The implication is that students' difficulties with the limit concept will likely affect their comprehension of other mathematical concepts and ideas that depend on it. On the contrary, students who have a nuanced understanding of the limit concept can easily solve limit-based calculus problems. The contention is that students should possess a nuanced understanding and knowledge of a limit concept to succeed in calculus.

Traditional teaching approaches characterized by routine learning and memorization of procedures often contribute to learners' challenges in the limit concept (Kado, 2021). The contention is that how the limit concept is taught and introduced to students has implications for their understanding of that concept. As a result, researchers explored the effect of technology-based instruction (Baye et al., 2021; Huda et al., 2025; Kado, 2021; Munyaruhengeri et al., 2025), which is believed to alleviate students' challenges and enhance their performance in the limit concept. While acknowledging the importance of such teaching approaches, this study does not seek to implement a particular teaching approach but rather focuses on student teachers' conceptions.

Furthermore, teaching and learning resources such as calculus textbooks often consist largely of many exercise problems limited to only evaluating the limit of a function at one point, which focuses on the domain process without clearly indicating the range process (Hong, 2023). In addition, students' difficulties with the limit concept are attributed to an inadequate algebraic knowledge and misconceptions (Jameson et al., 2023; Öztürk & Sönmez, 2020). The implication is that students may progress to pre-service mathematics education programs with an underdeveloped understanding of the limit concept. Such could imply that they may lack the prerequisite knowledge necessary for developing conceptions of a limit concept. Hence, Fernández et al. (2024) argue for the necessity of teacher education in developing not only their understanding but also their ability to notice prospective learners' thinking about this concept to promote mathematics achievement. In light of the central role of ITE in shaping student teachers' mathematical understanding (Saleh et al., 2025) and developing the next generation of teachers who can contribute to societal development (Simamora et al., 2024), it is important to explore their conceptions of the limit of a function.

While studies on student conceptions of the limit concept have been investigated in countries such as Austria (e.g., Ableitinger et al., 2023), Sweden (e.g., Juter, 2007), and Turkey (e.g., Duru, 2011), there are fewer studies in South Africa with a similar focus (Bansilal & Mkhwanazi, 2022; Bezuidenhout, 2001). One such study explored pre-service student teachers' conceptions of the notion of limit (Bansilal & Mkhwanazi, 2022). Inductive analysis of 74 student teachers' written responses revealed, amongst others, that "students focused on the function approaching a number but could not integrate this condition well enough with the variation in the x-values" (Bansilal & Mkhwanazi, 2022, p. 2096). Unlike Bansilal and Mkhwanazi’s (2022) study, this autoethnography study focuses on mapping out an individual student teacher’s conception of the limit of a function across different representations of the concept. These representations, as later described, include the numerical interpretation of a limit, the intuitive definition of a limit, the application of the intuitive definition of a limit using a table of values, and the graphical determination of a limit. This article agrees with Fernández et al.'s (2024) assertion that understanding the limit concept should be underpinned by different representation modes, such as numerical, graphical, and algebraic.

Sfard's (1991) dual nature of mathematical conceptions is used as a theoretical framework to analyze student teachers’ conceptions about the limit of a function. According to Sfard (1991), the process of learning is premised on “an intricate interplay between operational and structural conceptions” (p. 1). Firstly, in operational conceptions, a mathematical concept is conceived as a product of a computational process (Machaba, 2017; Sfard, 1991; Wedman & Bennet, 2025). As a result, the operational conception in this article is characterized by students’ procedures, mathematical actions, and algorithms associated with the limit of a function concept. Secondly, structural conception focuses on a mathematical concept that is conceived as a static structure where it can be referred to as if it were a real thing (Machaba, 2017; Sfard, 1991; Tossavainen & Helenius, 2024). Thus, for structural conception, we focus on student teachers’ visualization of the limit concept as if it were an abstract object. Thus, we use this framework to illustrate that the student teachers’ conceptions about the limit of a function concept can be conceived, both structurally and operationally. Although these constructs are seemingly incompatible, they are complementary (Sfard, 1991). Therefore, we consider students’ verbal descriptions and various kinds of visual representations as important for interpreting their conceptions of the limit concept.

In arguing from a psychological perspective, Sfard (1991) identifies three hierarchical stages that explain the transition from operational to structural conceptions in mathematics concept formation: interiorization, condensation, and reification. Firstly, the interiorization stage focuses on students’ ability to perform operations on lower-level mathematical objects (Sehole et al., 2023). In this regard, the focus is on student teachers’ ability to determine the limit of a function using a numerical approach. Secondly, condensation focuses on student teachers’ ability to alternate between various representations of the limit concept without feeling an urge to go into details (Sfard, 1991; Tossavainen & Helenius, 2024). Lastly, reification involves student teachers’ ability to conceive the limit of a function concept as a fully-fledged object. This stage can be characterized by student teachers’ proficiencies in evaluating the limit of a function by using various mathematical procedures, such as factorization. While this theory has been used as a lens to explore students' understanding and errors in mathematics concepts such as functions (Machaba, 2017; Sehole et al., 2023), literature is scarce on its application in the limit of a function. As a result, this study employs it to analyze student teachers’ conceptions of the limit of a function across various representations. The study sought to answer the following research question:

- What are the mathematics student teachers’ conceptions of the limit of a function?

## RESEARCH METHOD

This study adopted the autoethnographic research design, which Adams et al. (2017) describe as a “research method that uses personal experience (“auto”) to describe and interpret (“graphy”) cultural texts, experiences, beliefs, and practices (“ethno”)” (p. 1). It suffices to mention that autoethnographic studies often depart from traditional academic writing characterised by a third-person, passive voice and neutral, objective tone by writing in the first-person tone, as recommended by Cohen et al. (2018). As a result, our experiences as Introduction to Calculus module lecturers will help us describe and interpret our student teachers’ conceptions about the limit concept of a function. As we are directly immersed in our students’ learning experiences, we “can talk about these issues in ways different from others who have limited experiences with these topics" (Adams et al. 2017, p. 3). We consider it essential to explore the conceptions our students attach to the limit concept, as it will enable us to reflect on our practices. We contend that as teacher educators, we can inform outsiders about characteristics of our students' conception of the limit of a function "that other researchers may not be able to know" (Adams et al., 2017). Nevertheless, we are not necessarily implying that we can articulate our students' conceptions more accurately than outsiders.

In this autoethnography study, document analysis was used to systematically review student teachers' written responses to elicit in-depth meaning and develop empirical knowledge about their conceptions of the limit concept (Cohen et al., 2018). Thus, we use student teachers' written responses to a test as the main source of data. The test was administered as part of a formative assessment to first-year mathematics students enrolled in a 4-year Bachelor of Education degree at a public university in Limpopo Province. The test was based on the limit concept, covering the intuitive definition of a limit (students' understanding of limits as an abstract concept); the application of the intuitive definition of a limit using a table of values (students' ability to use tables of values to determine limits); the graphical determination of a limit (student ability to use graphs to illustrate limits); and the algebraic determination of a limit (students' proficiency in finding limits through algebraic manipulation). Purposive sampling was used to identify three students' scripts for analysis. Our rationale for selecting a grain-sized sample was in line with Andrade's (2021) recommendation that an analysis of a small number of participants can yield a wide range of insights into the phenomenon under investigation. The criteria for selecting the scripts included students who passed the test with high marks and middle marks and those who failed to achieve below 50%.

Thematic analysis was used to analyze the students' test scripts. It suffices to mention here that the test knowledge areas, as indicated above, were used as predetermined themes or a priori codes in the thematic analysis process (Braun & Clarke, 2006; Cohen et al., 2018; Taylor-Powell & Renner, 2003). The thematic analysis process consisted of numerous structured steps to ensure that the analysis was systematic and aligned with the research objectives. Firstly, the familiarization step involved thoroughly reading the test scripts to gain a general understanding of the content and context. This initial exposure was important for identifying overall patterns and understanding the students' expressions of their conceptions of limits. Secondly, the predetermined themes were derived from the aspects connected to calculus (e.g., intuitive understanding, numerical representation, graphical interpretation, and algebraic representation). Thirdly, each test script was systematically coded by identifying questions of the test that corresponded with the defined themes. Thus, we grouped the excerpts from the test scripts according to the predetermined themes. This facilitated a clearer comparison of how each theme was represented across sampled scripts. Lastly, extracts from the students' test scripts were presented as evidence to illustrate each theme as part of writing the findings. In addition, the findings were contextualized within the existing literature, drawing parallels with previous studies that highlighted similar conceptions or misconceptions related to limits. The research process followed is outlined in Figure 1 below.

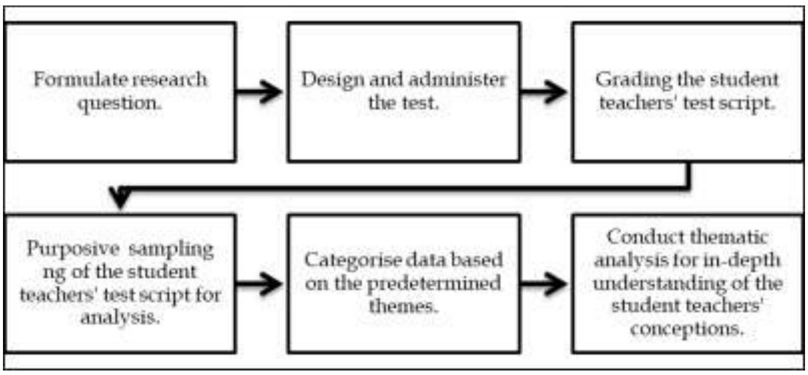


**Figure 1.** Flowchart of research procedure

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The test instrument was subjected to content validity through a moderation process conducted by a mathematics expert. The purpose was to ensure that the test adequately and accurately covers the domain items one intends to measure, as recommended by Creswell (2014). Student teachers' scripts were also moderated to ensure the validity of the results. Confirmability was achieved through using student teachers' script extracts in the analysis of the results (Cohen et al., 2018). Dependability was ensured through providing detailed methodological documentation of the research procedures and decisions made during the study (Ahmed, 2024). The following ethical considerations guided the study: Permission to use the students' test scripts was granted. Pseudonyms (e.g., Student Teacher A, Student Teacher B) were used in naming the extracts from the scripts to protect the identity of our students (Creswell, 2014).

## RESULTS AND DISCUSSION

The result of the student teachers' conceptions of the limit concept was divided into four themes, i.e., (1) Intuitive definition of a limit, (2) Application of the Intuitive Definition, (3) Graphical determination of a limit, and (4) Algebraic determination of a limit.

### Intuitive Definition of a Limit

Rather than asking our students to state the intuitive definition of a limit, we asked them to use their understanding of the definition to rewrite $\lim_{x \to b} g(x) = P$ in three different ways. A student who has some understanding of the definition should be able to see that as $x$ is made close to $b$ but not equal to $b$, $g(x)$ gets close $P$ or as $x$ gets closer and closer to $b$ but not equal to $b$, $g(x)$ gets closer and closer to $P$ or $g(x)$ can be made as close as you want by making $x$ close to $b$ but not equal to $b$. Although some student teachers' conceptions of this concept allowed them to make the three representations of the definition, most student teachers' responses raise questions about their conceptions of this concept. Below is a selection of a few responses to this question.

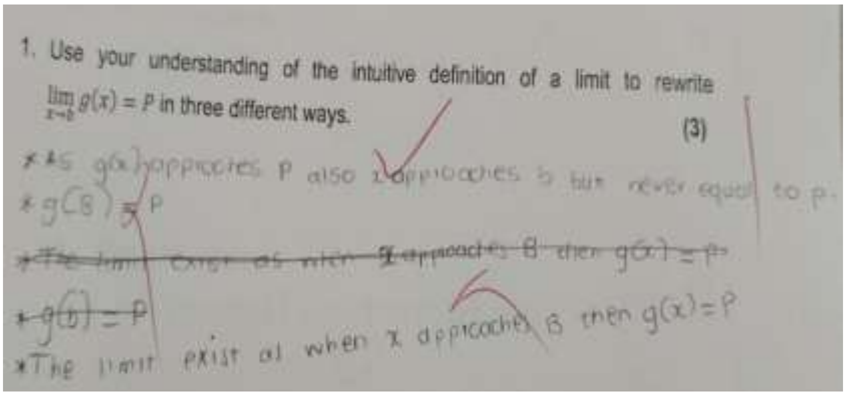
1. Use your understanding of the intuitive definition of a limit to rewrite $\lim_{x \to b} g(x) = P$ in three different ways. (3)

Figure 2. Student Teacher A's attempt at the intuitive definition

Based on Figure 2, Student Teacher A does not exhibit an understanding of the intuitive definition of a limit. The first sentence suggests that the student teacher does not know that the function is a dependent variable. The presentation of the definition suggests that x is a dependent variable. The student's response further indicates that he considers the limit of a function and the function as the same construct. Consequently, the result suggests that the student teacher's idea of a limit is not yet conceptualized. This affirms previous results on student confusion of a limit and function value, as reported in the literature (Adhikari, 2020; Bansilal & Mkhwanazi, 2022). Similarly, another student teacher responded as follows:

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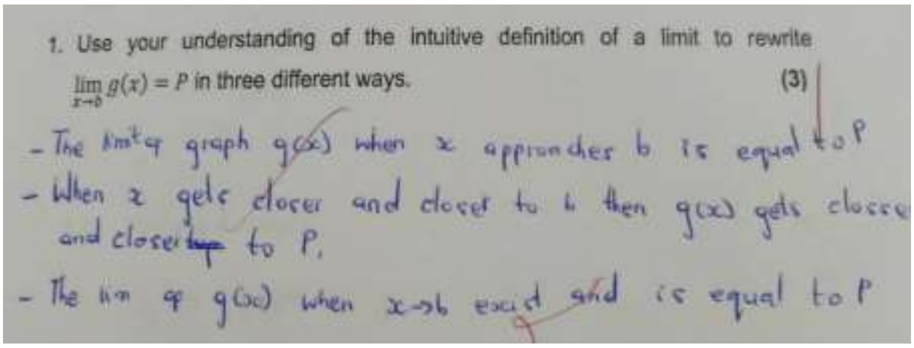
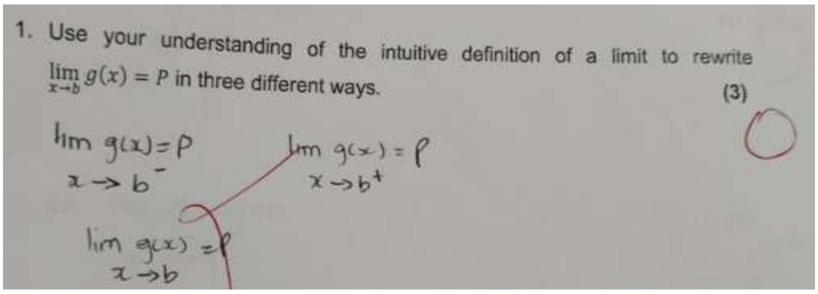

1. Use your understanding of the intuitive definition of a limit to rewrite $\lim_{x \to b} g(x) = P$ in three different ways. (3)

- The limt of graph g(x) when x approaches b is equal to P
- When x gets closer and closer to b then g(x) gets closer and closer to P.
- The lim of g(x) when x→b exist and is equal to P

**Figure 3.** Student Teacher B's attempt at the intuitive definition

Based on Figure 3, Student Teacher B's first response, the student teacher is simply "reading" the given statement. The third response talks about the existence of the limit. This response is not relevant to the question of the intuitive definition of a limit. This indicates that conceptions about the limit concept are incompatible with a concept definition, highlighting a lack of a nuanced understanding (Adhikari, 2020; Sebsibe, 2019). For this student, the idea of the limit is not yet conceptualized. The intuitive definition of a limit is anchored around what happens in the neighborhood of a dependent variable of a function as a result of what happens in the neighborhood of the independent variable of the function. Along the same line, another student teacher used the existence of a limit as shown below:


1. Use your understanding of the intuitive definition of a limit to rewrite $\lim_{x \to b} g(x) = P$ in three different ways. (3)

$\lim_{x \to b^-} g(x) = P$ $\quad$ $\lim_{x \to b^+} g(x) = P$

$\lim_{x \to b} g(x) = P$

**Figure 4.** Student Teacher C's attempt at the intuitive definition

As shown in Figure 4, the student teacher used his knowledge of the existence of a limit as one of the three ways an intuitive definition of a limit could be represented. Although the response is mathematically correct and aligns with some conceptions of a limit concept, the student teacher did not give a correct answer to the question. For this student teacher, it is evident that the student teacher has developed some understanding of what a limit of a function is; however, he could not apply such an understanding to the intuitive definition. Thus, the result echoes superficial understanding, where the student puts emphasis on the memorization of the rules without understanding them (Munyaruhengeri et al., 2024). Nevertheless, it would be interesting to see if this can consistently apply this rule to questions (e.g., graphical representation questions) that require such reasoning.

**Application of the Intuitive Definition of a Limit Using a Table of Values**

The table of values provides an opportunity for student teachers to apply their understanding of the intuitive definition of a limit. We are following the same students to see how they respond to a question based on a table of values.

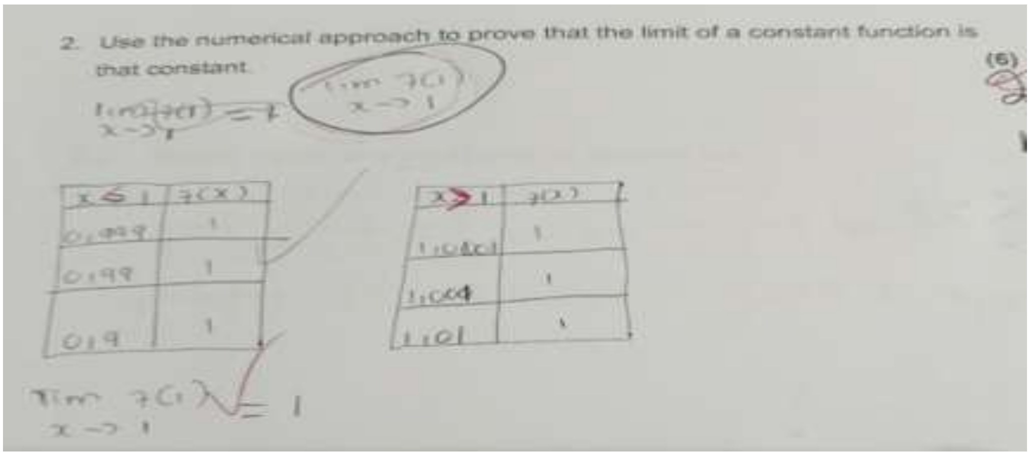



**Figure 5.** Student Teacher A's attempt at the table of values

Based on Figure 5, Student Teacher A could only give two tables of values. It is evident here that the student teacher could only give the part of the solution that focuses on procedures that involve the use of numbers only. Given his responses to the first question, one can argue that his lack of understanding of the intuitive definition of a limit made it impossible for him to give a correct answer to this question here. Hence, his response, $\lim_{x \to 1} f(1) = 1$ suggests that he consider $f(1)$ as a constant function. The two questions so far suggest that this student's conception of the limit concept is not developed. That he can develop a table of values does not talk about what a limit of a function is. It only suggests that there is some understanding of how to evaluate functions at various values of $x$. We proceed with Student B's attempt at the numerical approach as follows:

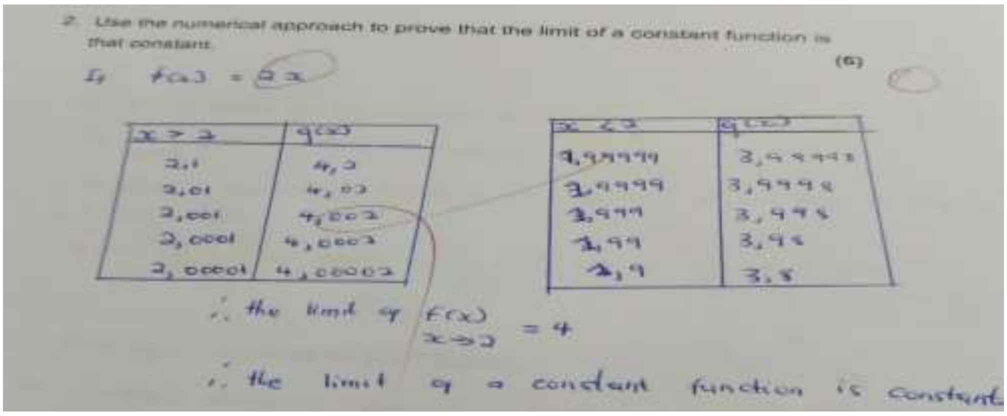



**Figure 6.** Student Teacher B's attempt at the table of values

Three critical skills are key to the development of the limit concepts. Student teachers should be able to 1) define and differentiate the types of functions, 2) evaluate functions for any value of x in the given domain and generate a table of values, and 3) analyse the behaviour of both the domain and the range using the understanding of the intuitive definition of a limit to determine the limit of the given function. Student Teacher B, as shown in Figure 6, does not demonstrate any of the three skills. For example, he considered $f(x) = 2x$ as a constant function, which indicates a lack of the first skill identified. This questions his conception of the limit concept, particularly of a constant function. The results affirm students' difficulties and misconceptions related to a function, as reported by Messias (2024) and Sebsibe 2019. We proceed with Student C's answers to the numerical approach as follows:

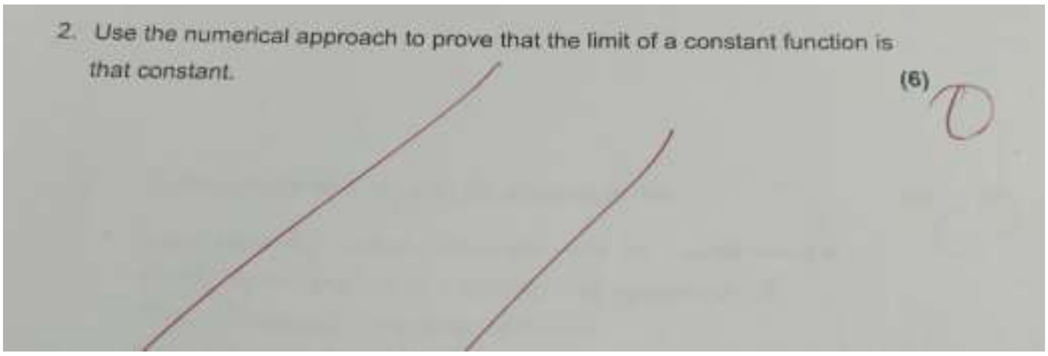



**Figure 7.** Student Teacher C attempt for the table of values

Based on Figure 7, student C did not respond to this question, suggesting a lack of understanding of using the numerical approach to prove the concept limit of a function. The only interpretation we can make, which is consistent with her response to question 1, is that she does not exhibit any understanding of the intuitive definition of a limit, and by extension, her conception of the limit concept is non-existent. However, our views on this student are juxtaposed against her responses to the graphical and algebraic representations of the limit concept in the next two sets of analyses. The result affirms the views of student difficulties on the limit of a function concept as reported in the literature (Assagaf & Arwadi, 2021; Bansilal & Mkhwanazi, 2022; Baye et al., 2021).

**Graphical Determination of a Limit**

To show that students have developed some conceptions of the limit concept, they should be able to read and determine the limits of functions from graphs of functions. Two important skills are required to determine a function's limit graphically: 1) the ability to interpret graphs of functions, and 2) the understanding of the intuitive definition of a limit. We use the third question based on the graph below (Figure 8) to continue to form some ideas on the three student teachers' conceptions of the graphical determination of a limit concept.

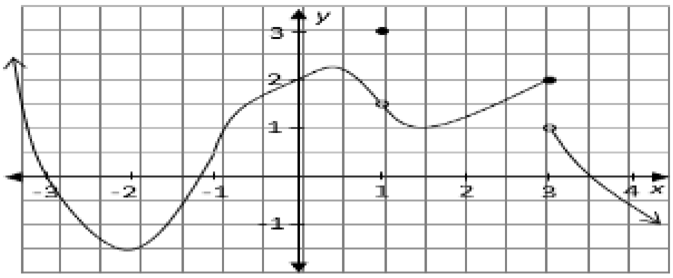



**Figure 8.** Graphical representation of a limit

As with the previous sub-sections, we focus on each student's responses as follows:

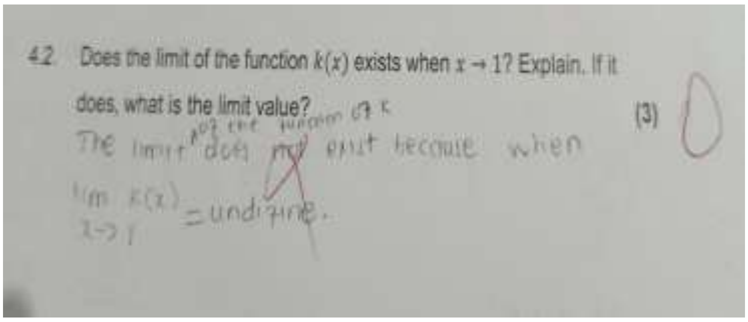
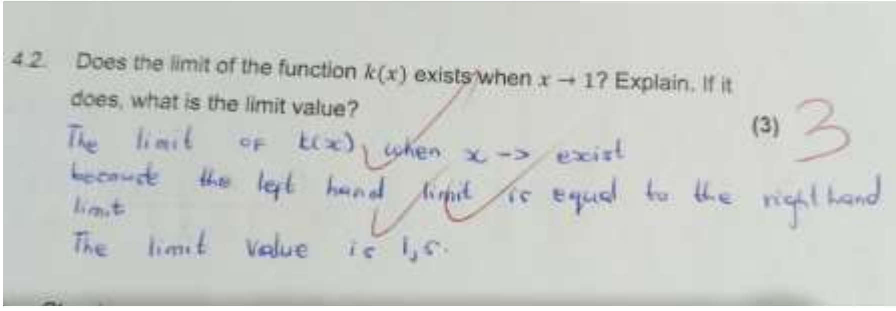



**Figure 9.** Student Teacher A's attempt at the graphical representation of a limit

Based on Figure 9, Student Teacher A's response to the question shows that he cannot read the graph to determine the function values. This student teacher's response suggests that if the function is discontinuous at a point, the limit of the function is undefined, which shows a lack of understanding of the conditions for a limit to exist. The result reveals a conceptual challenge, wherein the student teacher demonstrates the confusion between the function value and the existence of a limit. The assertion we are making aligns well with the assertions we made on this student teacher's response to the first two questions. Among possible explanations is that student teachers do not work well with questions that require them to give explanations rather than calculations or procedural algorithms. The result of the study supports previous studies that found that the students confused the function at a point and its limit from the graph (Adhikari, 2020; Messias, 2024; Munyaruhengeri et al., 2024). We proceed to Student B's answers for the graphical interpretation as follows:

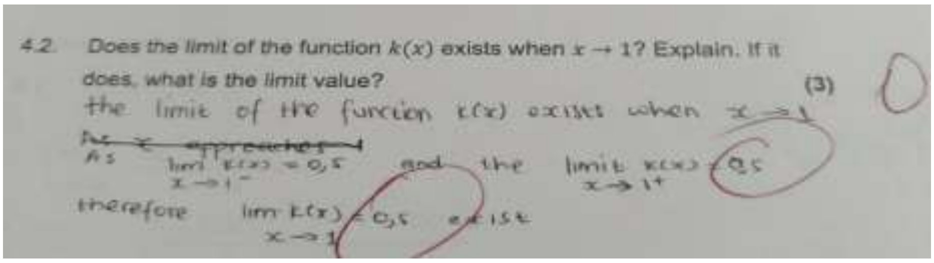

4.2. Does the limit of the function $k(x)$ exists when $x \to 1$? Explain. If it does, what is the limit value? (3)

The limit of k(x) when x -> exist because the left hand limit is equal to the right hand limit
The limit value is 1,5.

3

**Figure 10.** Student Teacher B's attempt at the graphical representation of a limit

Although we gave this student teacher full marks for this question, as indicated in Figure 10. Her response does not give more details on how she concluded that the left-hand limit is equal to the right-hand limit. Such a statement was given in their notes as a condition for the existence of limits. The results suggest that the student was able to interpret the graphical representation and determine the limit value without performing the necessary calculations. From Sfard's (1991) dual nature of mathematical conceptions, the student teacher's response indicates the condensation stage, where the student teacher does not consider the need for going into details. Thus, the result suggests that the student teacher has some idea of what a limit is and the conditions for it to exist. Similarly, Öztürk and Sönmez (2020) also revealed a high level of the pre-service teachers' graphical understanding of the concept of limit. We proceed to student teacher C's answers for the graphical interpretation as follows:


4.2 Does the limit of the function $k(x)$ exists when $x \to 1$? Explain. If it does, what is the limit value? (3)

the limit of the function k(x) exists when x -> 1
~~As x approaches 1~~
As $\lim_{x \to 1^-} k(x) = 0{,}5$ and the limit $\lim_{x \to 1^+} k(x)$ 0,5
therefore $\lim_{x \to 1} k(x)$ 0,5 exist

0

**Figure 11.** Student Teacher C's attempt at the graphical representation of a limit

Based on Figure 11, Student C is aware of the condition for the existence of a limit. However, he was unable to read the graph. As a result, he incorrectly considered the limit value to be equal to. At this level of study, it is worrying that this student cannot read graphs. A similar concern was also noted in Student A's response. Reading or interpreting graphs is a necessary skill in the study of calculus. The results indicate agreement with Sebsibe's (2019) earlier study, which found that most students lack knowledge of representing functions using different methods, such as

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graphical representation. However, this student does not show any conception of the limit concept, given that he has not responded positively to all the questions thus far.

**Algebraic Determination of a Limit**

The intuitive definition of a limit was used during our teaching to cultivate a feeling for the concept and, most importantly, to convey the idea of a limit intuitively. Once the idea of the limit concept was consolidated, we engaged student teachers in exercises that required them to apply algebraic manipulations to determine the limit of functions. This was done to expose our students to various ways the limit of functions is determined. We see this as another factor that explains the students' conceptions of the limit concept. Below is a selection of each student's responses to the algebraic determination of a limit question.

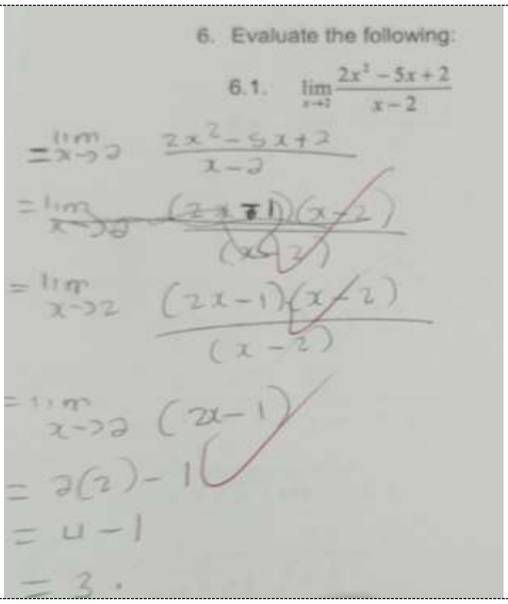


**Figure 12.** Student Teacher A. (Two figures have been placed side-by-side to show similar student answers)

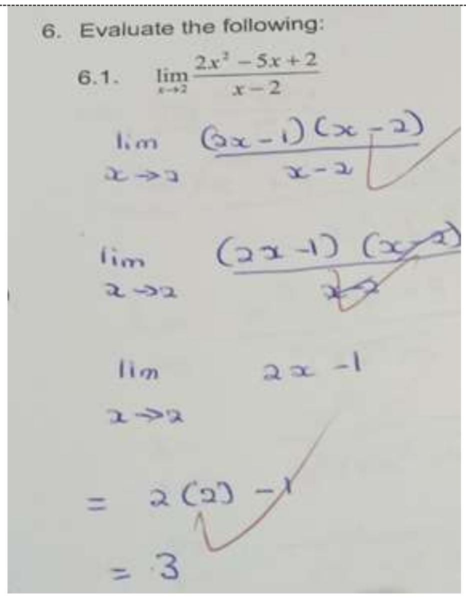


**Figure 13.** Student Teacher B. (Two figures have been placed side-by-side to show similar student answers)

Based on Figures 12 and 13, the results show that both student teachers A and B have a conception of algebraic representation of the limit concept. They applied the relevant algebraic manipulations by factorising the numerator and eliminating the like factors of the function represented by an algebraic fraction. Although student teacher A failed to determine the limit of functions through the numerical and graphical approaches, he managed to get the algebraic approach to the limit of a function correct. For student teacher A, the three approaches do not feed into each other. He treats the three approaches as separate, isolated entities of the limit concept. It is concerning that this student can only demonstrate some understanding of one of the four key skills that explain the student teacher's conception of the limit concept. Thus, this student teacher lacks the ability to move among various representations of the limit concept. From Sfard's (1991) dual nature of mathematical conceptions, both the student teachers possessed an operational conception as they demonstrated procedures and algorithms associated with the limit of a function concept. This is an indication that understanding the concept of limit only through algebraic representation does not necessarily suggest that this student has developed conceptions of the limit concept. Unlike student teacher A, student teacher B showed a correct conception of graphical representation but an underdeveloped conception of numerical representation. Nevertheless, it is satisfying to note that Student B is demonstrating some understanding of two of the four key skills that explain students' conception of the limit concept. The results not only uphold Sebsibe's (2019) earlier findings that students succeed in algebraic representation but also corroborate Bansilal and Mkhwanazi's (2022) notion that the students can succeed in some constructs but struggle with others related to the same concept. The next interpretation focuses on Student C's answers to the algebraic determination.

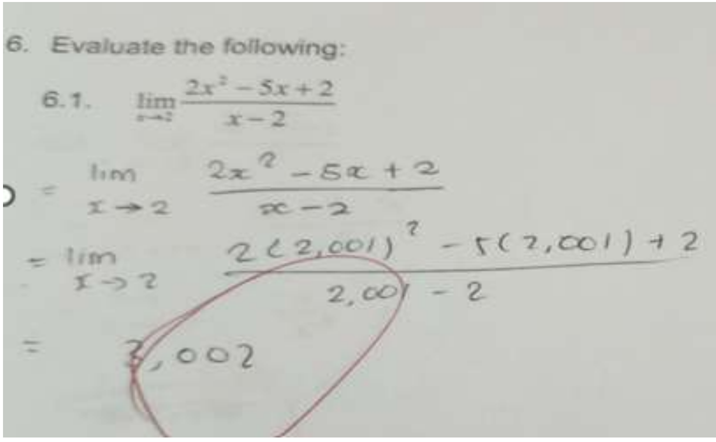


**Figure 14.** Student Teacher C's algebraic representation of a limit

Based on Figure 14, student teacher C applied a direct substitution for the value of $x$. This would result in an undefined quotient, and it would prompt the student to think of a different strategy. The result indicates that the student teacher cannot use algebraic knowledge to evaluate the limit concept. Thus, the student teacher demonstrated a lack of algebraic knowledge of simplifying quadratic fractions required in this question. Unlike the two student teachers discussed above, student teacher C's conception of the limit concept is non-existent or underdeveloped, as evident across various representations. The result is consistent with Jameson et al.'s (2023) and Öztürk and Sönmez's (2020) studies, which revealed students' challenges in determining the limit value through algebraic operations.

## CONCLUSION

The purpose of this article was to analyse the first-year mathematics student teachers' conceptions of the limit of a function. The findings revealed that the student teachers generally exhibit underdeveloped and fragmented conceptions of the limit of a function. Difficulties were not limited to the concept itself but extended to related foundational ideas embedded within it. The underdeveloped conceptions were noted on the graphical representation of the limit concept. Among possible explanations for difficulties in graphical representation is that some student teachers perceive the limit of a discontinuous function as undefined. Nevertheless, the findings revealed that student teachers generally perform well in an algebraic approach characterised by following rules and procedures rather than in other forms of representations, such as numerical and graphical representations of the limit concepts. The implications of the results point to some serious challenges that these three student teachers (and others with similar conceptions) are likely to experience as they continue their development of calculus concepts. As we indicated earlier, the limit concept is the foundation for future conceptions of calculus concepts. It is most worrying that they are unable to verbally, numerically, and graphically explain this concept. Over and above the importance of these skills in the development of other calculus concepts, we strongly believe that such skills are important in their pedagogical content knowledge development. Mathematics student teachers should possess these skills and other mathematical concepts to create meaningful learning opportunities for their prospective learners. The findings of this study cannot be generalised to a larger population of first-year mathematics student teachers in South Africa, since few student teachers' scripts from one university were analysed. In addition, this study employed document analysis as the only data collection instrument, which could be criticised on the issue of bias. It is recommended that future research focus on exploring, from multiple data sources, student teachers' transition difficulties across various representations of the limit concept.

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