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An Introduction of a CIRGON Geoboard as a Manipulative for Teaching Geometry in K-12 Mathematics

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ABSTRACT

South African learners consistently underperform in mathematics, particularly in geometry, as shown by national and international assessments such as the Trends in Mathematics and Science Study (TIMSS). Euclidean geometry remains a major area of difficulty, with learners performing significantly worse than in other mathematical domains, signaling persistent challenges in teaching and learning. A key contributing factor is the mismatch between the reasoning required for Grade 12 geometry tasks (typically at Van Hiele level 3) and learners' actual cognitive levels, which are often at Van Hiele level 1. This study adopts a practice-based research methodology to examine the effectiveness of the CIRGON Geoboard (CGG), a low-cost manipulative designed to support geometry instruction across the K–12 spectrum. Integrated into classroom practice and aligned with the Curriculum and Assessment Policy Statement (CAPS), the CGG promotes active, hands-on learning and scaffolds learners' progression through the Van Hiele levels of geometric reasoning. Findings from classroom-based interventions indicate that the CGG enhances learner engagement and conceptual understanding, helping to bridge the gap between curriculum demands and learner capabilities. The study concludes that the CGG shows promise for addressing persistent challenges in geometry education and recommends further research into its long-term impact on learners' geometric reasoning and problem-solving skills.

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INTRODUCTION

South African learners consistently demonstrate low achievement in mathematics in national and international assessments. The Trends in Mathematics and Science Study (TIMSS) specifically identified geometry content as a weakness for Grade 9 learners (Reddy et al., 2022). This concern is echoed in student perceptions, with studies indicating that learners attribute their difficulties in mathematics to challenges with Euclidean geometry (henceforth, geometry) (Bowie, 2009; Naidoo & Kapofu, 2020). Furthermore, analyses of learner performance in national examinations reveal consistently lower scores in geometry sections than in other mathematical content areas (Reddy et al., 2019; DBE, 2022). These findings align with earlier research by De Villiers (1996), who documented similar struggles among South African learners. Collectively, this body of research suggests a systemic challenge in developing a deep conceptual understanding of geometry and solving related problems (Naidoo & Kapofu, 2021). Consequently, South African Grade 12 learners consistently exhibit difficulties across various aspects of geometry and its application in problem-solving tasks.

Mosia et al. (2023) highlighted a critical disconnect between the demands of Grade 12 problem-solving tasks (requiring thinking skills on Van Hiele level 3) and the learners' actual geometric reasoning abilities, which fell predominantly within Van Hiele level 1. This finding aligns with prior research by Alex and Mammen (2016), who documented that Grade 10 learners primarily exhibited geometric thinking aligned with Van Hiele level 1. These results suggest a significant developmental gap between South African learners' actual geometrical thinking levels in the classroom and the Van Hiele levels they should be operating on in each grade. This highlights the need for instructional approaches that bridge this gap, potentially through targeted interventions or adjustments in teaching strategies to promote deeper understanding of geometric concepts before reaching higher-level problem-solving tasks. Furthermore, these results also raise questions about the alignment between assessment practices and Van Hiele levels. If Grade 12 problem-solving tasks require Van Hiele level 3 thinking, yet learners are predominantly operating at level 1, it suggests a mismatch between what is assessed and what learners are able to demonstrate.

The discrepancies identified between learner performance and curriculum demands necessitate the development of alternative teaching and learning experiences. These experiences should aim to develop learners' geometrical thinking to a level commensurate with the expectations outlined in the assessment standards of the Curriculum and Assessment Policy Statement (CAPS) (Reddy et al., 2022). Crucially, such instructional approaches should incorporate artifacts that are universally accessible and ensure equitable learning opportunities for all students, regardless of their socioeconomic background (e.g., low-cost manipulatives).

While existing manipulatives are often underutilized in South African mathematics classrooms, particularly in geometry, this paper introduces the CIRGON Geoboard (CGG)—a low-cost, modified geoboard designed for K–12 geometry education. Emerging from a larger PhD study that evaluated its effectiveness, the CIRGON board was designed to align with the Van Hiele theory of geometrical thinking framework to demonstrate its potential for scaffolding learner proficiency in geometry and aligning with CAPS assessment requirements. Thus, this paper begins by discussing the Van Hiele theory of geometrical thinking, and then we discuss manipulatives in plane geometry. Thereafter, we move on to discuss the geoboard as a manipulative in mathematics education and some readily available geoboards. Thereafter, we discuss a short methodology section that is followed by a detailed explanation of how and why the modifications of the suggested CIRGON Geoboard were effected. Modifications to the CIRGON Geoboard—proposed to better serve K–12 geometry—are introduced, and its alignment with the phase teaching guidelines outlined in the CAPS document for geometry is discussed. This study addresses the question: How can the CGG be used to scaffold learners' proficiency in geometrical thinking?

The Van Hiele theory of geometrical thinking was developed by Dutch educators Dina van Hiele-Geldof and Pierre Van Hiele in the 1950s and posits that students' progress through distinct levels of understanding in geometry is characterized by qualitative shifts in their reasoning and perception of geometric concepts. The Van Hiele proposed five geometrical levels of thinking, but only four are relevant for the purposes of this study. These levels are explained below:

- Level 1 (visualization): Students focus on recognizing and identifying shapes. Manipulatives like pattern blocks, tangrams, and virtual manipulatives (e.g., online apps) allow students to explore and identify shapes in both physical and digital environments. This multimodal approach strengthens spatial reasoning skills (Gutiérrez, 2008).
- Level 2 (analysis): Students begin to analyse properties of shapes and relationships between them. Manipulatives can be used to demonstrate these relationships in a tangible way. For instance, students can connect cubes to form rectangular prisms or use geoboards to discover angle measures and their connections.
- Level 3 (informal deduction): Students move towards formal reasoning and logical arguments. While manipulatives become less central at this level, they can still serve as visual aids to support the development of geometric proofs and justifications.

- Level 4 (deduction): Students can go beyond merely identifying the characteristics of figures, for instance. They can also construct proofs using definitions and postulates (Mason, 2009). Therefore, the theorem's proof should only be presented at this level (Abdullah & Zakaria, 2013). At this level, the student has mastered the theorem's visual aspect, its properties, and its application. The student can now investigate why they can use the theorem in greater depth.

Further extensive research revealed that there is a prerecognition level before visualization (level 0) where children use declarative knowledge to learn geometry (Clements, 1999). The pre-recognition level represents the earliest stage in the Van Hiele theory of geometrical thinking. At this level, children have a very basic understanding of geometry. They are unable to consistently identify and differentiate shapes based on their specific characteristics. Their geometric thinking is primarily perceptual, relying on visual impressions rather than conceptual understanding (Clements, 1999). For example, a child at the pre-recognition level might not consistently recognize a rotated square as the same shape due to variations in its appearance. Research highlights the importance of providing rich, varied experiences with shapes to support children's progression beyond this stage and into higher levels of geometric thinking (Clements & Sarama, 2020).

The Van Hiele theory emphasizes that effective geometry instruction should be aligned with the student's current stage of thinking, incorporating appropriate pedagogical strategies to facilitate advancement. Recent studies continue to support the relevance of the Van Hiele model in educational practices, highlighting its importance in curriculum design and teacher training to enhance students' geometric understanding and reasoning skills (Fuys et al., 1988; Yi et al., 2020). For example, Yalley et al. (2021) highlight the importance of the curriculum in improving learners' geometrical thinking from the pre-recognition level to Van Hiele level 2. This means that, if instruction is based on the development of geometrical thinking instead of the mere solution of a certain class of problems, then learners' performance may improve.

The teaching and learning of mathematics have dominantly involved the use of manipulatives as concrete tools for mediating teaching and learning by visually representing abstract mathematical concepts. With the rapid advancement of technological tools, manipulatives have recently been digitized, and mathematical concepts are represented and manipulated on a digital screen instead of using concrete objects (Hurst & Linsell, 2020). These may include digital software such as GeoGebra®, Geometer's Sketchpad®, Wolfram Alpha, and Cabri Geometry. Both virtual and physical manipulatives are based on visualization, allowing for the accommodation of average, below average, and above average students (Mthethwa, 2015, p. 45). This paper focuses on manipulatives as concrete and tangible artifacts that can be used to represent abstract geometrical concepts. These manipulative artifacts have been shown to enhance student learning and behavior (Galleto & Descallar, 2016), enhance student knowledge, support the integration of knowledge and skills, and contribute to overall student development (Sani & Salahudeen, 2016).

Early studies in plane geometry (or Euclidean geometry) have indicated that manipulatives play an important role in learning in the early years. The Van Hiele model of geometric thought development considers manipulatives as crucial tools for promoting students' progress through the hierarchical levels of geometric understanding. These concrete objects allow students to progress from basic visualization (Level 1) to formal reasoning (Level 3) through a process of exploration and abstraction. Even in the age of digital learning tools, manipulatives remain a cornerstone of effective geometry instruction according to the Van Hiele model. These physical objects serve as crucial stepping stones, facilitating students' progress through the hierarchical levels of geometric understanding (Fujita et al., 2017). By engaging with manipulatives, students can bridge the gap between concrete experiences and abstract concepts, fostering a deeper comprehension of geometric principles.

In plane geometry, manipulative tools such as geoboards can enable students to physically interact with and visually represent several geometric concepts and figures, ultimately solidifying these concepts as tangible and practical (Adelabu et al., 2019). Scholarly literature has further indicated that manipulative tools such as geoboards can effectively alleviate students' difficulties with Euclidean geometry, thereby enhancing their performance in related concepts and problem solving (Abari & Andrew, 2021; Deborah et al., 2020).

The geoboard is a mathematical manipulative used in educational settings. Geoboards are tools that can be used to explore fundamental concepts in plane geometry, including area, perimeter, and polygon properties (Scandrett, 2008). It consists of a physical board with an array of half-driven nails around which elastic bands (often referred to as geo bands) can be stretched (Wikipedia, https://en.wikipedia.org/wiki/Geoboard). Invented and popularized by Egyptian mathematician Caleb Gattegno (1911-1988), the geoboard serves as a versatile tool for exploring various geometrical concepts (see Figure 1).

These concepts include plane shapes, transformations (translation, rotation, reflection), similarity, spatial coordination, counting, right angles, classification of shapes, scaling, congruence, area, and perimeter (Gattegno, 1971). For example, students can create different plane figures, such as triangles or different quadrilaterals, and investigate their specific properties (Branfield, 1970; Rowan, 1990).

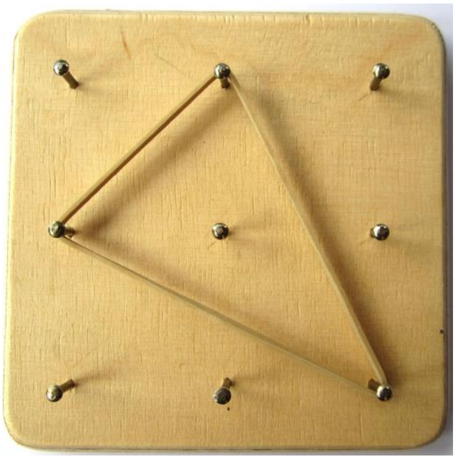

Figure 1. An example of an early Geoboard

Abdu-Raheem and Oluwagbohunmi (2015, p. 161) have highlighted a paradoxical situation where teachers express dissatisfaction with the lack of manipulative tools yet are failing to use tools that are readily available to them. This is evident in geometry, where most teachers do not use geoboards for teaching. Instead, they rely on the traditional chalk-and-talk approach, where students are presented with pre-worked-out examples and are expected to reproduce similar methods during examinations (Golafshani, 2013; Hurst & Linsell, 2020; Mogari et al., 2009). However, traditional chalk-and-talk teaching approaches may cause difficulties when students encounter geometry problems that are different from the ones they learned in class. Moyer and Jones (2004) further identified a knowledge gap among educators regarding the effective use of manipulative tools in the classroom. This lack of proficiency, they argue, contributes to some teachers justifying the omission of manipulatives from their instruction by citing unforeseen downtime during lessons. Some educators believe that manipulative tools can lead to cognitive confusion (McClung, 1998), especially if these tools do not meaningfully contribute to the overall effectiveness of teaching. Consequently, there has been a decline in the adoption of manipulative tools in the teaching of mathematics, as teachers do not know how to use and manage these tools for effective teaching and learning (Marshall & Swan, 2005).

An examination of relevant literature indicates the existence of square and circular geoboards. Square geoboards are the most common type and consist of a square or rectangular board with nails arranged in a square grid. These square geoboards come in various sizes and are appropriate for introducing basic geometric shapes, perimeter, area, and symmetry. Circular geoboards have nails arranged in a circular pattern, offering a unique platform for exploring circles, angles, and even fractions. Please refer to Figures 2 and 3 below for visual illustrations.

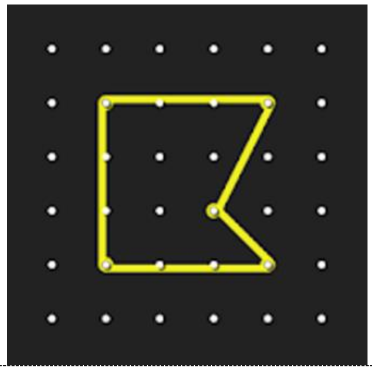

Figure 2. Square geoboard

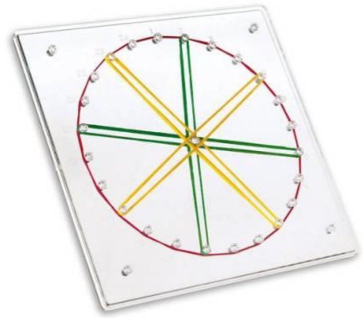

Figure 3. Circular geoboard

The use of virtual geoboards has increased as the internet has become more accessible. Virtual geoboards can be particularly useful for students who are easily distracted by physical manipulatives or those who have limited access to physical resources. The use of available geoboards (square, circular, and virtual) in the teaching and learning of mathematics has strengths and weaknesses. Below are some of the advantages of these manipulatives:

- Geoboards help to develop students' understanding of geometry.
- Geoboards can be used to teach geometry, measurement, and numeracy (Scandrett, 2008).
- Geoboards are used to explore fundamental concepts in plane geometry, including area, perimeter, and properties of polygons (Rahmiati, 2016).
- These readily available geoboards offer valuable support for student-centered learning, rendering them adaptable to both large classroom environments and individual study settings at home.
- The available geoboards serve as suitable and pertinent tools for fostering self-directed learning (SDL) by promoting student confidence, motivation, active participation, interest, engagement, and collaboration (Acharya, 2017).
- Geoboards are used as tools to improve students' attitude, achievement, and participation when learning geometry concepts.
- Also, geoboards provide students with the freedom to learn on their own (Sibiya, 2020)
- The use of geoboards has a positive effect on students' performance in Euclidean geometry (Deborah et al., 2020).
- Geoboards encourage active participation in the classroom (Sani & Salahudeen, 2016; Rahmiati, 2016).
- Students may use a variety of tools like geoboards to reason, make connections, and solve problems (Srirman & Pizzuli, 2005).

While the strengths of available Geoboards are aligned with the basic education aims from grades R–12, it seems that existing Geoboards have several limitations:

- The construction and measurement of tangent lines—which are required in circle geometry (Grades 10–12) as well as other geometry concepts across various grade levels—are impeded by the absence of spaced pins. These readily available Geoboard designs are unable to facilitate the construction of theorems that involve tangent lines (such as the tan-chord theorem), tangents meeting at the same points, tan-radius theorems, and proportionality theorems.

- These readily available geoboards are restricted to specific grade levels. For instance, the square geoboards are exclusively utilized for concepts of space and shape (geometry) that are taught from Grade R–9; the circular geoboards are used for Euclidean geometry concepts in Grades 10–12 (DBE, 2011).
- The existence of two different Geoboards used at different grade levels may cause a financial burden, and students may think that concepts of space and shape (geometry) and concepts of Euclidean geometry are unrelated.

These limitations have underscored the necessity for the development of a modified Geoboard to address these limitations. Looking at these limitations, the decision was made to design a geoboard that can be used for all geometry-related concepts at all grade levels. This paper reports on the characteristics of such a geoboard, provides guidelines for using this geoboard in the classroom, and furnishes some examples of how this manipulative can be used for teaching and learning.

RESEARCH METHOD

The study utilized practice-based research methodology—methodology used to bridge the gap between research and practice (Candy et al., 2021)—to create a geometry teaching and learning tool for K–12 mathematics. The created output for teaching geometry in K–12 mathematics is called a CIRGON Geoboard (CGG), a Geoboard that was created as an improvement from the previously existing Geoboards, which had several grade limitations as highlighted in the discussion in the previous section.

RESULTS AND DISCUSSION

The Modified Geoboard (The CIRGON Geoboard (CGG))

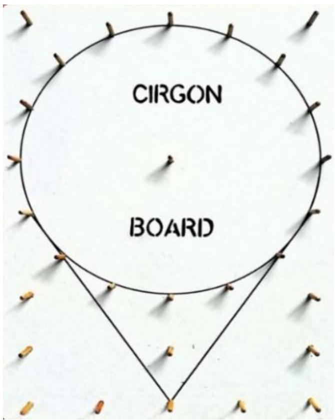

Figure 4. The CIRGON geoboard

This paper introduces the CGG, a manipulative tool designed to facilitate the teaching and learning of fundamental geometric concepts. Unlike traditional geoboards, the CGG boasts versatility, allowing its application across a broad spectrum of grade levels, from primary and secondary schools to tertiary institutions. This adaptability stems from its design features. Constructed from wood with a rectangular shape, the CGG utilizes nails evenly spaced at 1 cm intervals to create a grid. These nails serve as anchors for elastic bands or strings, enabling students to construct various geometric shapes and explore their properties visually. Additionally, integrated rulers and protractors affixed directly on the board facilitate precise measurement of lines and angles. The CGG was also designed with a circle in the middle of the rectangular board to accommodate circle geometry theorems and for investigating any geometrical problem that involves circles. The CGG is further complemented by a dedicated manual containing activities specifically designed to enhance Euclidean geometry instruction. The name 'CIRGON' itself reflects this versatility: it combines 'circle' and 'polygon' to symbolize the board's capacity for constructing both curved and straight-sided shapes. A visual of the CGG is presented in Figure 4.

The key innovation of the CGG lies in its expanded functionality compared to existing Geoboards. While traditional models often cater to specific grade levels or geometric concepts, the CGG's design allows for the exploration of a wider range of geometry principles across various educational stages. This versatility aligns with the findings of Ndioho and Olajide (2020), who emphasized that geoboards are effective tools capable of encompassing a wide range of activities that cover nearly all core geometry concepts. By offering a comprehensive and adaptable manipulative, the CGG has the potential to enhance geometry instruction across the curriculum and support learners as they progress through increasingly complex mathematical ideas. A key finding of this study is the CGG's capacity to enhance student engagement with Euclidean geometry theorems by enabling the hands-on construction of geometric figures in diverse shapes and sizes. This observation supports Carrolla's assertion (as cited in Scandrett, 2008) that geoboards assist learners who experience difficulty in sketching figures, making it easier for them to explore and verify geometric properties. Through tactile interaction and guided exploration, students can move beyond rote memorization to meaningful understanding.

The CGG also facilitates a visual and interactive approach to understanding theorems involving circles and straight lines—such as the well-known theorem stating that "*the angle subtended by a diameter is a right angle*." Students can construct this theorem in multiple configurations, thereby reinforcing their understanding of its generality and applicability. This result is consistent with findings from Laski et al. (2015), who found that manipulative tools enhance learners' comprehension of abstract and complex mathematical ideas. Similarly, Freire et al. (2018) reported that the use of geoboards contributed to students' deeper appreciation and conceptual grasp of Euclidean geometry, findings mirrored in this study's use of the CGG. The visual nature of the CGG allows students to not only construct but also validate geometric theorems. For example, Figure 5 illustrates the construction of a semicircle with a diameter and an inscribed angle, accompanied by a protractor that confirms the angle measure as 90°. This form of visual proof strengthens students' conceptual understanding, providing tangible confirmation of theoretical knowledge. Visualization, as numerous scholars have argued (Makina, 2010; Patahuddin et al., 2020; Presmeg, 1986, 2006), is a critical element in effective mathematics instruction. It supports students' problem-solving abilities and fosters deeper cognitive engagement (Carden & Cline, 2015; Haas et al., 2023; Kohen et al., 2022; Mudaly & Narriadoo, 2023).

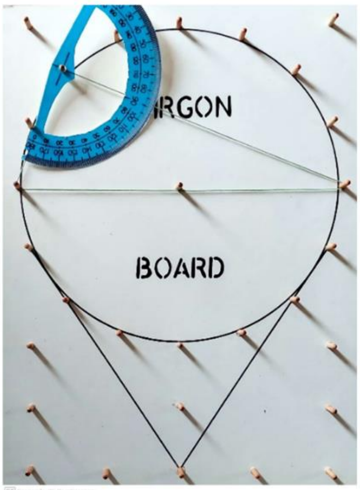

Figure 5. Measuring angles on the CGG Geoboard

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Beyond the demonstration of individual theorems, the CGG's ability to generate a wide array of geometric figures supports the investigation of relationships and properties, thereby enriching students' experience with Euclidean geometry proofs. Through the integration of visualization, tactile engagement, and cognitive reasoning, the CGG emerges as a powerful instructional tool capable of addressing persistent challenges in geometry education and advancing learners' understanding of spatial relationships.

Thus, using such a CIRGON Geoboard when teaching geometry does not only help students visualize geometrical theorems but also confirms through measurements that the theorem is always true. The CGG gives students opportunities to construct different sizes of the same geometrical object. This can allow for the teaching and learning of mathematical concepts such as congruency and similarity. This flexibility provides students with multiple representations of the same concept, which strengthens conceptual understanding. Furthermore, sustained student engagement with the CGG promotes active participation, which may positively influence learners' attitudes toward Euclidean geometry. These findings are consistent with those of Rahmiati (2016), who reported that geoboards enhance students' confidence, ability, and competitiveness in constructing plane shapes. To ensure the construct validity of the CGG and the accompanying Euclidean geometry worksheets, the materials were submitted for review by three mathematics education experts from a university. The experts critically evaluated the instructional design and posed probing questions regarding the CGG's features and applicability. Following an in-depth discussion with the researcher, consensus was reached that the CGG was conceptually sound and well-suited for the study's objectives. This peer validation process also contributed to establishing the reliability of the CGG, as it confirmed that the instrument could consistently support the intended instructional outcomes in various classroom contexts.

Some Guidelines for using the CGG for Teachers and Students

The implementation of the CIRGON Geoboard (CGG) was guided by the principles of practice-based research methodology, which focuses on improving educational practices through iterative design, reflection, and classroom-based evidence. In this study, the CGG was developed and refined through continuous cycles of planning, action, observation, and reflection within authentic teaching environments. The aim was not only to investigate its effectiveness as a pedagogical tool but also to evolve its design and application through the lived experiences of both teachers and learners. As part of this methodology, practical guidelines were developed to ensure the effective integration of the CGG into classroom instruction. These guidelines were informed by real-time teaching scenarios and refined through feedback loops with teachers and students. The following protocols were established to standardise and optimise the use of the CGG:

- Keep CGG empty before using it
- Ensure different coloured rubber bands are available as tools for creating two-dimensional plane geometry shapes
- Protractors and a ruler should be available to measure the sizes of angles and lengths of the sides of geometrical figures
- Specify the dimensions of the plane geometry shape that the students will create
- Stretch and connect the rubber bands to the CGG pins to create the intended shape
- Diagrams must always be clearly and accurately drawn on the worksheet.

In line with the progressive, constructivist orientation of practice-based research, teachers are encouraged to introduce geometric concepts using simple figures such as triangles, quadrilaterals, and circles, etc. Teachers should also go beyond constructing simple shapes as the learning progresses and should challenge the students to create pictures on their CGGs by means of the different colored rubber bands. Furthermore, the teacher should have the students create a design of any kind using at least 10 colored rubber bands. Then have them look more closely: what shapes can they find in the design? How many triangles are there? What kind of triangles are they? They will be surprised at what they find. Teachers can also encourage deeper thinking by having students make shapes by using other shapes, such as making a rectangle using only triangles. As mentioned earlier, these are some of the primary guidelines for using the CGG for teaching and learning. This exploration promotes both visualization and geometric reasoning, key goals within the Van Hiele framework and the CAPS curriculum.

Teachers are further encouraged to stimulate higher-order thinking by assigning tasks that require constructing new shapes from existing ones—such as creating a rectangle from multiple triangles. These activities not only support the acquisition of geometric knowledge but also foster student agency, creativity, and collaborative problem-solving. Ultimately, the implementation of the CGG, embedded in practice-based research, allows for authentic experimentation, critical reflection, and ongoing refinement of both tool and pedagogy. The adaptability and structured flexibility of the CGG make it a sustainable and scalable innovation for enhancing geometry instruction across grade levels.

A Description of a Euclidean Geometry Lesson with the CGG

Even though it was argued that the CGG could be used in Grades R–12, this section describes how it can be used to teach a Euclidean geometry theorem, which, in the South African context, is applicable to Grades 10–12. The lesson was on the geometry theorem, '*the line segment joining the center of a circle to the midpoint of a chord is perpendicular to the chord*,' and the students were third-year pre-service teachers (PSTs) preparing to teach mathematics to Grades 8 – 12.

The third-year PSTs were given a ruler (or straight edge), a protractor, rubber bands, and the CGG. The example from this section was taken from a series of Euclidean geometry lessons taught by means of the CGG. This specific section was from week 4, when learners were being taught circle geometry theorems. During week 4, students learned about the properties of circles, and they were asked to complete the proof of the theorem stating that '*the line segment joining the center of a circle to the midpoint of a chord is perpendicular to the chord*.' The general pedagogical approach was based on active learning, where students followed instructions from a designed worksheet where they investigated certain properties by utilizing the CGG.

Students were instructed to construct Euclidean geometry diagrams on a modified geoboard using colored elastic rubber bands and to measure the angles with a protractor—exactly as instructed on the Euclidean geometry worksheet.

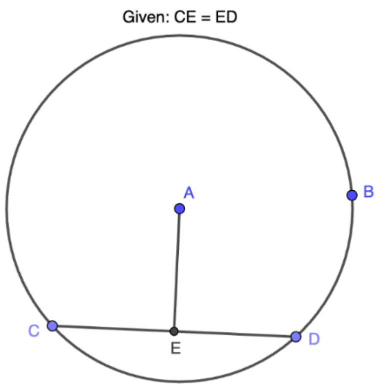

Figure 6. Theorem 1 from the CGG manual

The students were first required to reproduce the diagram shown in Figure 6 on the CGG, and it was reiterated that they had to use the ruler to ensure that $CE = ED$. Once this task was completed and verified by the instructor, the students were required to accurately measure the angles and the lengths of specified sides. They were required to measure the sizes of $\angle AEC$ and $\angle AED$ and the lengths of CE and ED respectively. The students concluded that $\angle AEC = \angle AED = 90°$.

CONCLUSION

In conclusion, the CIRGON Geoboard (CGG) emerges as a promising tool for interventions aimed at addressing the shortcomings in geometry education that have been identified in South African schools. By offering a versatile and accessible manipulative tool, the CGG has the potential to bridge the disconnect between the curriculum demands and learners' Van Hiele levels. This conclusion is supported by the CGG's design features that cater to a broad spectrum of geometric concepts across all grades. Furthermore, the CGG aligns with the CAPS assessment guidelines and promotes active learning through hands-on exploration. Future research can investigate the effectiveness of the CGG in real-world classroom settings and its impact on learners' geometric reasoning abilities and problem-solving skills. The limitation of the research is that pre-service mathematics teachers who participated in the experimental group were not given extensive training on how to use the CGG to construct different geometry figures. The time taken to implement the intervention (six weeks) is another limitation to this study because if the intervention had been implemented in a much shorter or longer period, the findings might have been different from the current findings.

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