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Algebraic Question Types in Grade 10 Mathematics Textbook: A Cognitive Perspective

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ABSTRACT

Mathematics education depends on textbooks as the primary source of information for both teachers and learners while providing essential tasks that develop cognitive skills. The types of tasks used in these resources directly affect learners' cognitive growth and show why textbooks must contain various question types. Current research lacks examination of mathematics textbooks in South African schools, even though learners consistently perform poorly in mathematics. This study investigated whether a widely used Grade 10 Mathematics textbook challenged learners to various question types that help build problem-solving skills and high-level thinking. A deductive content analysis approach evaluated 1 360 questions from the textbook's algebra content, including algebraic expressions, exponents, equations and inequalities, number patterns, and functions. The research results indicated that most questions (62%) required routine calculations while the content lacked questions that required higher-order cognitive skills such as problem-solving, argumentation and interpretation. The assessed textbook appears to fail learners by not adequately developing critical thinking skills that learners need for both future advanced math classes and practical applications. The study recommends more diverse question types in mathematics textbooks to develop learners' cognitive skills and problem-solving abilities.

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INTRODUCTION

Textbooks are essential in all education systems, helping to translate curricular objectives into classroom practices. They are crucial resources for teachers, mainly in subjects like mathematics, where the quality of textbook content can significantly impact learners' cognitive development (Knight, 2015). Numerous studies including those by Van De Ham and Heinze (2018), Tekkumru-Kisa (2020), Agterberg (2021), and Mita (2021), have emphasised the importance of textbooks in shaping learners' thinking skills, mainly through the types of questions posed and the cognitive demands these questions entail (Van De Ham & Heinze, 2018; Tekkumru-Kisa et al., 2020; Agterberg et al., 2021; Mita et al., 2021). In South Africa, research shows that most teachers relay on textbooks for lesson planning and delivery (Leshota, 2020). The literature (Gracin, 2018; Pratama & Retnawati, 2018; Fan *et al.*, 2013; Dole & Shield, 2008) reveals a prevalent issue in mathematics education: textbooks often fail to sufficiently challenge learners, focusing mainly on lower-order cognitive skills while neglecting higher-order thinking (Gracin, 2018; Pratama & Retnawati, 2018; Fan et al., 2013; Dole & Shield, 2008).

The trend is problematic for learners because it raises questions about how textbooks help learners develop critical thinking and problem-solving skills that they need to succeed in life and in their future careers (Star *et al*., 2015). Research has also established that there are variations in the cognitive load of the textbooks used in different countries. For instance, Kul (2018) designed a study that compared mathematics questions in Canadian and Turkish textbooks; the results indicated that Canadian textbooks provided more questions that demanded higher cognitive load although both countries had similar knowledge dimensions. In addition, Zhu and Fan (2006) reviewed the lower secondary grade mathematics textbooks from the United States and China to determine the type of tasks that were open-ended, application-based, traditional, or routine in nature. The study found that both contexts had a lot of routine tasks with very little application to real life. In line with this, Pratnawati and Retnawati (2018) observed that textbooks may not entirely support the growth of 21st-century competencies that focus on the higher-order thinking tasks.

In the context of algebra—a subject crucial for developing logical reasoning and abstract thinking—many studies have pointed out that learners struggle significantly, often due to the way algebra is presented in textbooks (Romli, 2016; Samuel *et al*., 2016). Algebra's abstract nature requires learners to go beyond rote memorisation and engage in deeper cognitive processes, such as recognising patterns and applying abstract principles to solve complex problems (Arcavi *et al*., 2017). However, research by Pournara (2016) indicates that learners frequently make errors in algebra due to a lack of foundational understanding, which is exacerbated by textbooks that do not adequately support the transition from arithmetic to algebraic thinking.

In addition, numerous studies such as Vincent and Stacey (2008) and O'Keeffe and O'Donoghue (2011) have criticised textbooks for contributing to what has been called the "shallow teaching syndrome," which is characterised by the emphasis on procedural knowledge as opposed to conceptual understanding. Although this approach is more convenient to implement it does not do a great deal to encourage the growth of higher order thinking skills such as problem solving, critical thinking and the ability to make connections between different mathematical concepts.

Despite extensive research on textbook quality, there remains a significant gap in the literature regarding algebra content in South African Grade 10 Mathematics textbooks, particularly concerning the cognitive levels and types of questions presented. Existing studies have largely focused on international contexts, leaving a void in understanding how South African textbooks align with the cognitive demands of the curriculum. This gap is particularly concerning given the critical role of algebra in developing logical reasoning and abstract thinking, which are foundational for advanced mathematical learning and application (Grønmo, 2018), and the persistent learners' poor achievement in mathematics in benchmark examinations, especially on questions demanding higher-order thinking skills (Diagnostic Report, 2023). According to various studies, the absence of higher-order questions in textbooks suggests an untapped opportunity to enhance learners' cognitive development (Pournara *et al*., 2016; Romli, 2016; Samuel *et al*., 2016).

This study aims to fill this gap by exploring the cognitive levels and types of questions in a widely used Grade 10 Mathematics textbook in South African schools. The objective was to scrutinise the cognitive levels and types of algebra questions in the textbook. The expectations are that the results will be useful to textbook authors, teachers, and policymakers as they strive to enhance the quality of mathematics education so as to enable learners to deal with the cognitive demands of higher education and other post-secondary activities.

THEORETICAL FRAMEWORK

This article adopted a theoretical framework approach to analyse the types of algebraic questions and their cognitive levels in the Grade 10 Mathematics textbook. The framework is based on two primary sources: Gracin's (2018) analytical framework for mathematical activities and the

Cognitive levels as prescribed in the Curriculum and Assessment Policy Statement (CAPS) in South Africa (Department of Basic Education, 2020). When combined, these sources create four major dimensions for assessing mathematical tasks: representation and modelling, calculation and operation, interpretation, and argumentation and reasoning. These dimensions are also related to the cognitive levels identified in CAPS which include Knowledge (K), Routine Procedures (RP), Complex Procedures (CP), and Problem Solving (PS) to ensure that the analysis of the content of the textbook is thorough. The following is a graphical representation of the integrated theoretical framework used in this paper as shown by Figure 1.

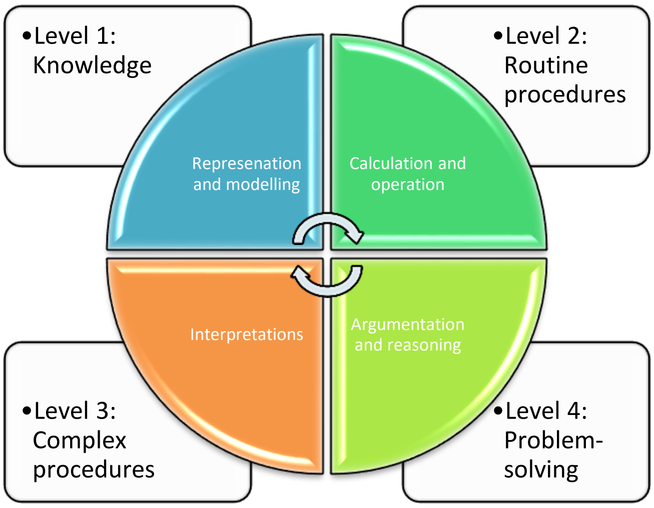

Figure 1. Eight-dimensional framework outline

Gracin's Dimensional Models

This framework divides mathematical activities into four dimensions: representation and modelling, calculation and operation, interpretation, and argumentation and reasoning. These dimensions offer a systematic way of analysing the nature of algebraic questions to understand their cognitive demands in detail.

Representation and Modelling

Representation (H1) in mathematics refers to the process of translating mathematical information into alternative expressions such as graphing data or writing equations or using other symbolic representations. Modelling goes beyond this by using mathematical relationships to solve problems which represent real world situations thus creating a mathematical model of the problem. The function $t(x) = k^x + q$ is described by the following properties:

- $k > 0; k \neq 1$
- X – intercept at (3 ; 0)
- The horizontal asymptote is $y = -8$

a) Determine the equation of $t(x)$.

b) Sketch the graph of $t(x)$. Show clearly the intercepts with the axes and asymptotes.

Learners are given properties of the function $t(x) = k^x + q$ and are required to transform this information into multiple mathematical forms. They begin by interpreting the symbolic information (e.g., intercepts, asymptotes) and converting it into an equation. Next, they transform the equation into a visual representation by sketching the graph, showing the intercepts and asymptotes. This process exemplifies *Representation* because learners use the function's algebraic form to represent it visually and symbolically. This dimension is crucial for assessing how well textbooks engage learners in applying mathematical concepts to practical situations, as emphasised by Gracin (2018).

Calculation and Operation

The execution of basic mathematical operations on real or abstract numbers is referred to as Calculation (H2). This dimension encompasses tasks like solving equations, performing unit conversions and conducting elementary geometric constructions. For instance, solve for x, given: $\frac{3}{5}(2x+7)-3=\frac{5x-2}{3}$. The task of solving the equation is included, which means that learners are required to perform several algebraic operations such as distributing, combining like terms and isolating the variable x. Operations that are basic to the dimension of *Calculation* are those that involve manipulating abstract numbers through processes such as addition, subtraction, multiplication and division. The task also shows the need of learners to practice these operations in order for them to get a better picture of how algebraic procedures work. This dimension is crucial for assessing how effectively textbooks are able to help learners build procedural fluency which is one of the key aspects of mathematical proficiency (DBE, 2020).

Interpretation

Interpretation (H3) entails understanding and contextualising mathematical information presented in various forms, such as graphs, symbols, or tables. It requires learners to make sense of mathematical representations and relate them to the problem context. Thus, learners are encouraged to communicate their ideas through graphs and symbolic expressions (Salami1 & Spangenberg, 2024). For example, consider the graphs of $f(x)=-x^2+4$ and $g(x)=x+2$ are sketched in Figure 2. B and C are the x – intercepts of $f(x)$. A and D are the y – intercepts of $f(x)$ and $g(x)$ respectively. $f(x)$ and $g(x)$ intersect at B and E.

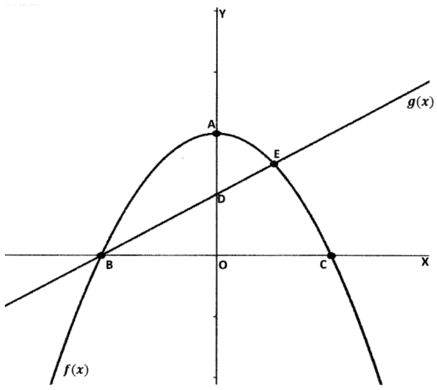

Figure 2. The graphs of $f(x)$ and $g(x)$

a) Write down the coordinates of A.
b) Determine the coordinates of E, a point of intersection of $f(x)$ and $g(x)$.
c) For which values of x will: $f(x)<g(x)$.

In the example above, the learners have to relate the graphical representations to the equations and use this information to answer questions like "For which values of x is $f(x)<g(x)$?" This task needs them to interpret and understand the mathematical data in its context, and to determine where one function is below the other from the graph. All in all, this example demonstrates how textbooks can help learners to interpret and link mathematical representations to enable a better comprehension of the problem's context. Gracin (2018) notes this as a critical part of mathematical proficiency since it moves scholars beyond basic computation to include an appreciation for how various mathematical representations connect to one another. This dimension addresses the extent to which textbooks enhance understanding and the capacity to extract important conclusions from mathematical information (Gracin, 2018).

Argumentation and Reasoning
Argumentation (H4) involves the logical explanation of mathematical concepts to support or refute a particular position. Reasoning extends this by engaging learners in constructing sound mathematical arguments that lead to conclusions. For example, Tabelo is currently four times as old as his daughter, Linda. Six years from now, Tabelo will be three times as old as Linda. Calculate Linda's age currently. To solve the problem, learners need to construct a logical argument, using the relationships provided to find Linda's current age. First, they define variables for the unknown ages, then reason step by step through the problem, solve the equations, and justify their conclusions. This process requires a clear explanation of how they arrived at the solution, demonstrating their understanding of the relationships between the variables. This example demonstrates how textbooks can foster learners' ability to create sound mathematical arguments and apply logical reasoning to solve complex problems, as highlighted by Gracin and the DBE (2020).

Cognitive Levels
The Department of Basic Education (2020) defined four cognitive levels together with their weightings for assessment for example examinations: Knowledge 20%, Routine procedures 35%, Complex procedures 30% and Problem solving 15%. These levels correspond to and are guided by the categories in the 1999 TIMSS taxonomy, which offers a standardised way of defining the cognitive complexity of textbook question assessments.

Knowledge – L1: This level involves recalling factual information or basic concepts.
Routine Procedures – L2: In this level, learners use procedures they have learned to solve basic problems. A situation which demonstrates this might involve solving a quadratic equation using a well-known formula which shows more procedural understanding than conceptual understanding. For example, solve for m: $(m-1)(m-1)=0$. The quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ can be used to solve for m. This example aligns with the *Routine Procedures – L2* cognitive level because it focuses on the mechanical application of a familiar method to solve a quadratic equation, without requiring extensive conceptual thought. The task helps evaluate how well learners can follow and execute routine processes in mathematics.

Complex Procedures – L3: These tasks require more intricate reasoning and a deeper understanding of mathematical concepts. For example, simplify:

$$\frac{1-2x}{4x^2-1}-\frac{x+4}{2x^2-3x+1}+\frac{1}{1-x}$$

The example is comparable to Complex Procedures – L3 because it implies several algebraic operations, deeper or arterial conceptual understanding, and the capacity to reason through a multistep process – all of which are more complex than basic or routine procedures.

Problem-Solving – L4: At this level, learners are expected to think critically and come up with solutions to complex problems through analysis, evaluation and creation. For example, if the pattern 'MATHSMATHSMATHS' is repeated this way, what will the 267th letter be? This does not solve simply follow a formula or apply a standard procedure. Learners need to think about the repeating pattern, understand it, and come up with a plan how to find out which letter is the 267th. This entails a close examination of the pattern and the sequence of the events in the pattern. This example is comparable to Problem-Solving – L4 cognitive level of thinking because it requires learners to engage in higher order thinking, which includes analysing and evaluating the structure of a repeating sequence and coming up with a solution through careful reasoning. It is not a routine or complex procedure but rather a strategic one for solving a non-routine problem.

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Linking Gracin's Dimensions to Cognitive levels

Dimension 1: Representation and modelling linked with level 1: Knowledge

At L1, the focus is made on the recognition and use of basic representations like tables, equations and graphs. In Gracin's representations and modelling dimension, learners recall and apply foundational knowledge to change information into different forms like writing equations or sketching graphs from given data, showing familiarity with mathematical concepts.

Dimension 2: Calculation and Operation Linked with level 2: Routine Procedures

In H2 learners apply familiar mathematical techniques, like solving equations to quite simple problems. Such tasks require the execution of known procedures, without complex reasoning, which is procedural fluency.

Dimension 3: Interpretation Linked with Level 3: Complex Procedures

The H3 dimension involves understanding and contextualising mathematical data, which requires more intricate reasoning. This links to L3 as learners must analyse and interpret the relationships between different forms of data, like equations or graphs, and apply deeper understanding to solve multi-step problems. In the H3 dimension learners are required to understand and humanize mathematical data which is more detailed and demanding. This is related to L3 since learners have to comprehend the connections between various data representations, such as equations and graphs, and use this understanding to solve complex, multiple-step problems.

Dimension 4: Argumentation and Reasoning Linked with the Level 4: Problem-Solving

In the H4 dimension, learners are expected to develop and justify mathematical arguments based on logic, which aligns with the problem-solving type of thinking. This level requires learners to analyse, evaluate and synthesise information in order to develop and justify solutions to complex, non-routine problems, which helps in the development of critical thinking and reasoning skills.
Virtually all the questions have been classified based on the four dimensions of mathematical activities and their cognitive levels to determine the accuracy of the textbook implementation based on the CAPS objectives and the development of learning cognitive skills. Therefore, this approach not only assists in the identification of the strengths and weaknesses of the textbook and how it can be improved but also assists in the identification of how it prepares learners for the higher order thinking in mathematics which is vital in the real life. First, the integration of Gracin's (2018) framework with the DBE's cognitive level taxonomy ensures that the analysis is both comprehensive and relevant to the context, thus making it a valuable contribution to the current discourse on the quality of mathematics education in South Africa. This article focuses on algebra a central part of the mathematics curriculum and thus addresses a significant gap in the literature and offers practical recommendations for improving textbook design and instructional practices.

RESEARCH METHOD

In this article, the purposive sampling design was used. In qualitative research, this approach is used to increase the efficiency of the limited resources by selecting situations that can produce much information as recommended by Patton in 2002, as cited in Palinkas (2015). Purposive sampling is a process of identifying and choosing participants or data sources who are very familiar with the topic of the study (Creswell & Clark, 2011 as cited in Palinkas *et al*., 2015). For our research aim, the Mind Action Series Mathematics textbook for Grade 10 was chosen as the main study material. The first was based on the relevance of the material, its conformity to the CAPS curriculum and the approval by the Department of Education. Only one topic from this textbook was analysed, and that is Algebra, which is one of the most challenging subjects for secondary mathematics learners. Since this study was based on the analysis of a textbook, there was no need to have a research site in the form of human subjects. The textbook was the main research subject and provided an insight into its content, structure and pedagogical elements. The lack of research site ensured that focus and rigour were maintained with no contamination from the environment.

In our study document analysis was used as the main data collection method. Document analysis is defined as the process of analysing different forms of documents both in hard copy or electronic format (Bowen, 2009). Like other qualitative research approaches, document analysis is a process of studying and making inferences from data in order to gain understanding and develop theory (Corbin *et al*., 2008 as cited in Bowen, 2009). The Mind Action Series Mathematics textbook for Grade 10 was used as the single source of data. This textbook was chosen because it is relevant to the South African education curriculum and it is used in most secondary schools. The textbook was systematically analysed to determine the content, organisation and the level of challenge compared to the CAPS curriculum.

The inclusion and exclusion criteria for the data were: area of study had to include Algebra, the CAPS guidelines had to be followed, the participant had to be of different cognitive level and the question types had to be distributed across the sections. An Excel spreadsheet was used as a structured document analysis instrument. The spreadsheet had four essential columns: number of the question, type of question, level of cognitive, and remarks. This made it easy to enter data into the database and sort the data in a systematic way. To ensure the accuracy of the analysis, the research instrument (Excel spreadsheet) was pre-tested with a chapter of the book to ensure the correctness of the analysis. This ensured that the process of categorisation was well established and data was entered in the right manner. The process of data collection involved the following: choosing the textbook, determining the Algebra chapter, creating the data collection tool, extracting and organising data, applying analytical frameworks, checking for consistency and consolidating the findings.

The focus of the analysis was placed on the Algebra chapter of the Mind Action Series Mathematics textbook for Grade 10. Due to the fact that the data was qualitative, deductive content analysis was used. According to Gracin (2018), DBE (2020) content analysis is a qualitative method of analysing and comprehending the content of a text. Two analytical frameworks were used to guide the analysis: The framework of Gracin (2018) in classifying question types, which are representation, calculation, interpretation, and argumentation, and the Cognitive Levels of questions by DBE (2020) which include knowledge, routine procedures, complex procedures, and problem solving. Both the authors of this paper together with the researcher analysed every question and then assigned it to those frameworks. Where possible, remarks were made to offer additional explanations. The outcomes were then compiled together to ensure accuracy, and any discrepancies were discussed and addressed to produce the final categorisation.

In the research process, a structured approach was adopted. First, a textbook was chosen and then the Algebra chapter was identified for examination. An Excel spreadsheet was created as a data collection tool to systematically pick and quantify the data. With the help of analytical frameworks, question types and cognitive levels were categorised. Then data was checked again and compared and discussed to make sure that the data was accurate and then finalised for consolidation and interpretation. Thus, the research was conducted in a very systematic and scientific manner as informed by the research objectives and which provided useful insights into the pedagogical efficacy of the material.

RESULTS AND DISCUSSION

The analysis of Grade 10 Algebra content covers topics including algebraic expressions, exponents, number patterns, equations and inequalities, and functions. A summary of these findings is presented in Table 1.

Table 1. Mathematics topics analysis.

Topics	Question Type	Total	Cognitive level	Total
Algebraic expressions	H1	64	L1	121
	H2	376	L2	290
	H3	34	L3	73
	H4	11	L4	1
Total		**485**		**485**
Exponents	H1	0	L1	124
	H2	265	L2	131
	H3	0	L3	12
	H4	3	L4	1
Total		**268**		**268**
Equations and Inequalities	H1	23	L1	29
	H2	154	L2	129
	H3	0	L3	16
	H4	2	L4	5
Total		**179**		**179**
Number patterns	H1	0	L1	5
	H2	71	L2	62
	H3	0	L3	4
	H4	0	L4	0
Total		**71**		**71**
Functions	H1	103	L1	50
	H2	200	L2	272
	H3	0	L3	33
	H4	54	L4	2
Total		**357**		**357**

The analysis of the content reveals a noticeable imbalance in the distribution of question types and cognitive levels across several algebraic content topics, including algebraic expressions, exponents, equations and inequalities, number patterns, and functions. Most of the questions are concentrated at the lower cognitive levels, such as "Routine Procedures" and "Knowledge," which collectively dominate the textbook's content. This imbalance raises significant concerns about the extent to which the textbook supports the development of higher-order thinking skills among learners.

In the algebraic expressions topic, 60% (290/485) of the questions fall under the "Routine Procedures" category, requiring learners to apply established algorithms without engaging in higher-order thinking. *For instance, page 4, exercise 1(d), learners must show that the following recurring decimals are rational, (1).* $0,\dot{4}$ (2). $0,\ddot{2}\dot{1}$ (3). $0,1\dot{4}$. This requires recalling the concept of rational numbers and performing straightforward calculations. Only 15% (73/485) of these questions involve "Complex Procedures," *for instance, page 5, example 4(d), learners must determine between which two consecutive integers the irrational number* 2π *lies,* requiring more conceptual understanding. Furthermore, only 0.2% (1/485) of the questions fall under "Problem-Solving." *An example from page 31, a consolidation and extension exercise, requires learners to show that the sum of the reciprocals of two number is equal to* $\frac{1}{2}$*, given that their product is 40 and their sum is 20.*

This trend continues in the exponents topic, where 46% (124/268) of the questions are categorised under "Knowledge". *For instance, page 33, exercise 1(a), requires learners to simplify expressions such as (3).* $2^5 \times 2^3 \times 2$ *and (19).* $\frac{x^5}{x^3}$. This requires recalling exponent laws. Another 49%

(131/268) fall under "Routine Procedures." *An example from page 34, exercise 1(d), asks learners to simplify expressions such as (1).* $12x^3 - 3x^3 \times 2$ *and (2).* $2x^3 \times x^2 + 3x \times x^4$, requiring basic procedural calculations. Only 5% (12/268) are "Complex Procedures," and just 0.4% (1/268) involve "Problem-Solving," as seen on page 40, where learners must show that $b = \frac{a}{a-1}$ *if* $a = 1 + 2^n$ *and* $b = 1 + 2^{-n}$. Notably, 99% of the questions in this topic are purely computational, with no interpretation, representation, or argumentation required.

The trend continues in the equations and inequalities topics, where 72% (129/179) of the questions are classified as "Routine Procedures," *for instance, page 46, example 1(b), solve the linear equation* $\frac{4x-1}{3} - \frac{7x+2}{6} = \frac{x}{2}$. The learner is required to apply the Brackets, Orders, Division, Multiplication, Addition, Subtraction (BOMAS) rule as a procedure to solve this problem since the equation contains fractions, and must insert brackets around the numerators if they are expressions of more than a single term, then determine the lowest common denominators (LCD). There were only 9% (16/179) requiring "Complex Procedures," *for instance, page 49, exercise 2(b), solve the equation* $(x-3)(x+4) - (3x-2)^2 = x - 8x(x-1)$, and 3% (5/179) involving "Problem-Solving," *for instance, page 55, exercise 5(c), at ABC shop, 3 hamburgers and 2 cooldrinks cost R89, whereas 2 hamburgers and 3 cooldrinks cost R73,50. Determine the cost of 1 hamburger.*

In the number patterns topic, an overwhelming 87% (62/71) of the questions are categorised under "Routine Procedures, *for instance, page 42, example 1, 6; 10; 14; 18; ……… is a given sequence, (a). determine the general rule (nth term), (b). calculate the 100th term.* Only 6% (4/71) fall into the "Complex Procedures" category, *for instance, page 45, consolidation and extension exercise, the learner is required to determine the 999th and 1000th terms, if the sequence given is: 1; 3; 1; 6; 1; 9; 1; 12; 1; …… and none designed for problem-solving.* This shows that "learning aimed at practicing problem-solving skills falls under the purview of complex cognitive learning" (Ghofur, Jatmiko & Sanjaya, 2023).

The functions topic shows a similar pattern, with 60% (290/357) of the questions under "Routine Procedures, *for instance, page 109, exercise 1(a). Given:* $y = x, y = \frac{3}{2}x, y = \frac{1}{4}x$, *which of the three lines is the steepest? Explain.* Only 15% (73/357) fall under "Complex Procedures," *for instance, page 136, exercise 5(b). Sketch the graphs of the following functions on different axes, (8).* $y = (3-x)(3+x)$, *and* (9). $y = 3^{-x} + 3$ and a mere 0.3% (2/357) is dedicated to "Problem-Solving," *for instance, page 162, consolidation and extension exercise, g (6), the possible values of* t *if the graph of* $f(x) = -3x^2 + 12 - t$, *(i) does not cut or touch the x-axis, (ii) cuts the x-axis in two distinct points.*

The dominance of lower-order cognitive tasks indicates that the textbook prioritises procedural fluency instead of conceptual understanding and critical thinking. The findings of this study are in sync with the research conducted by Vincent and Stacey (2008) and O'Keeffe and O'Donoghue (2011) which state that most of the mathematics textbooks focus on procedural knowledge rather than constructing a deep understanding of the subject. The absence of problem-solving tasks in key algebraic topics is most critical as algebra provides a foundation for advanced mathematical reasoning (Arcavi *et al*., 2017; Romli, 2016). Kul *et al*. (2018) and Zhu and Fan (2006) have shown that across the globe, textbooks focus on routine procedures, thus restricting the learners' scope for higher order thinking. This leads to a 'shallow teaching syndrome' (Gracin, 2018) where learners can apply procedures but have no understanding of the concepts behind them.

The limited presence of questions that demand higher cognitive engagement as well as questions about complex procedures or interpretation or argumentation possibly indicates that the textbook may not be sufficient for learners to meet the cognitive requirements of the curriculum or for the development of higher-level mathematical thinking. The selected textbook shows a limited number of higher-order tasks which coincides with the findings of Kul (2018) and Zhu and Fan (2006) who point out that most textbooks give priority to routine procedures at the expense of critical thinking and problem-solving skills. This imbalance may create a "shallow teaching syndrome" which makes learners good at procedure implementation but fails to develop their critical thinking and conceptual understanding (Gracin, 2018).

The mathematics curriculum should contain various cognitive tasks in order to help learners achieve procedural fluency as well as problem-solving and reasoning skills (DBE, 2020). According to Pournara (2016), the concentration on calculation-based questions restricts the development of essential skills such as analysis, reasoning and justification of mathematical concepts among learners. In algebraic topics, it is significant to understand abstract structures and relationships. Stein (2000) and Hiebert (2005) have shown that the involvement of learners in complex problem-solving tasks enhances their concept generalisation skills. Boesen (2014) also established that learners who were given non-routine problems showed better mathematical reasoning as compared to those who were given only procedural practice.

To better prepare learners for higher-order cognitive challenges, textbooks should incorporate a greater variety of question types. For instance, increasing the availability of complex procedures and problem-solving tasks in algebraic expressions and functions would be in line with the recommendations made by Kilpatrick (2001). Also, incorporating tasks that ask for argumentation and representation would help in the development of critical thinking skills. According to the NCTM (2014), Schoenfeld (2013) have recommended a problem-based learning, where learners apply mathematical reasoning in solving problems as opposed to memorising procedures. The study by Li (2019) indicated that learners who learned from textbooks with diverse cognitive tasks performed better than those who learned from traditionally presented materials. These results indicate that shifting the distribution of question types in algebraic content topics may benefit learners' mathematical proficiency in the long run.

This article also reveals a critical deficiency in the development of the Grade 10 Mathematics textbook. The current concentration on the lower order cognitive skills and the traditional procedures together with the limited number of higher-order thinking tasks shows that there is a need to change the current textbook design approach. Future editions of the textbook should have more questions that are in the form of interpretation, argumentation and problem solving. These changes will help to meet the international best practice and will ensure that learners not only possess procedural fluency but also critical thinking skills that are necessary for success in academic and working life (Organisation for Economic Co-operation and Development, 2018).

CONCLUSION

The study shows that most of the questions, in the Grade 10 Math book require calculations and do not challenge learners mentally or offer chances for problem solving skills to develop independently and think critically like they should in math learning process. These results could impact how we design curriculums and teach math in school because if textbooks do not include problem solving tasks it might lead to learners focusing much just procedural fluency without truly understanding the concepts involved in mathematics learning. In order to tackle this problem effectively and enhance learning outcomes in algebra education settings it is crucial for authors of textbooks and teachers to include a range of activities that emphasise critical thinking and logical reasoning skills. By incorporating life problem solving situations into the curriculum learners can not only gain a better understanding of the subject but also sharpen their analytical abilities. Nonetheless it is important to note that this research is constrained by its emphasis, on analysing textbooks than directly observing classroom teaching methods or learners' progress. The future researchers could investigate how teachers enhance textbook lessons by incorporating problem solving tasks and evaluate how updated textbook content affects learners' interest and academic performance over time. Furthermore, conducting studies may offer understanding on how modifications, in question formats shape learners' mathematical growth, in the long run.

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