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Secondary School Learners' Self-Efficacy and Achievement in Non-Routine Mathematics Problem-Solving

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ABSTRACT

This paper uses a correlational research design to investigate the predictive power of the sources of self-efficacy and non-routine problem-solving achievement. The data were collected from a sample of 118 high-achieving secondary school learners in the Gauteng province, South Africa, using questionnaires, a problem-solving test, and interviews. Their self-efficacy beliefs were categorized into mastery experience, vicarious experience, social persuasion, and physiological state. The findings revealed that the physiological state, vicarious experience, and mastery experience self-efficacy beliefs positively impacted problem-solving achievement, while that of social persuasion was negative. The magnitudes of the impact of the physiological state and social persuasion sources of self-efficacy were relatively greater than the others. Each source of self-efficacy was discovered to have an insignificant, weak, positive linear relationship with problem-solving achievement.

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INTRODUCTION

The ability to solve non-routine problems is of paramount importance in everyday life. Literature reports many factors that positively relate to mathematics problem-solving achievement, of which mathematical self-efficacy is among them (Voica et al., 2020; Dagdag et al., 2020; Amri et al., 2020; Wahidaturrahmi & Hayati, 2020; Yurt & Sunbul, 2014). Yurt and Sunbul (2014) view self-efficacy as an individual's belief that she/he can improve the desired learning behavior and outcome levels. Similarly, Fonna and Mursalin (2018) define mathematics self-efficacy as a learner's feeling, trust, and confidence in his/her ability to solve mathematics problems. All the definitions of self-efficacy seem to concur with one's judgment of his/her capacity to succeed in mathematics. Mathematics self-efficacy might exist in learners at various levels, ranging from low to high. A learner's level of self-efficacy might have a bearing on his/her self-perception as a mathematician, problem-solving behavior, and mathematics problem-solving achievement. Most papers (e.g., Arifin et al., 2021a; Agbornu & Edekor, 2020; Setyowati et al., 2019; Amri & Widada, 2019) argue that the higher the mathematics self-efficacy, the higher the mathematics problem-solving achievement. For instance, Setyowati et al. (2019) discovered that learners' levels of self-efficacy correlated positively with their learning outcomes. In their research, the learners with elevated levels of self-efficacy scored high in mathematics learning outcomes.

Sahendra et al. (2018) and Fonna and Mursalin (2018) related self-efficacy to some components of problem-solving, e.g., mathematical modeling. They discovered that the higher a learner's self-efficacy level, the greater the number of representations he/she uses in problem-solving. This means that self-efficacy possibly impacts the learner's approach to problem-solving. Additionally, Fonna and Mursalin (2018) observed that learners with high self-efficacy aimed

high and strived hard to achieve and be successful in mathematics learning. Self-efficacy has observable effects on a learner's actions, effort, persistence, flexibility, and self-actualization, just to mention a few. For instance, learners with high self-efficacy levels could display more confidence and persistence in solving challenging tasks (Voica et al., 2020). Pointedly, SE might limit the effort and time a learner devotes to problem-solving. On the same note, Adegoke (2015) characterizes learners with high self-efficacy as having good self-regulated learning. Considering this discussion, this paper highlights the following attributes as some of the indicators of self-efficacy: self-confidence in self-regulated learning, self-confidence in cognitive capacity or potential, and self-confidence in realizing self-actualization. It is important to improve learners' mathematics self-efficacy and problem-solving ability. Improvement in mathematics achievement can enhance the learners' self-efficacy. On this note, Wahidaturrahmi and Hayati (2020) maintain that learners who achieve high proficiency in mathematics tend to have high self-efficacy. As such, past and current mathematics problem-solving achievements may affect a learner's self-efficacy as he/she judges him/herself based on these experiences.

The mathematics self-efficacy beliefs emanate from four main sources: mastery experiences, vicarious experiences, social persuasions, and physiological or affective states (Bandura, 1977). The mastery experiences involve a learner's evaluation and interpretation of his/her past achievements in problem-solving. The past problem-solving experiences that a learner interprets positively might stimulate the development of positive self-efficacy beliefs to engage in more problem-solving activities (Ozcan, et al., 2021). The vicarious experiences involve a learner's evaluation and interpretation of his/her competencies or achievement in problem-solving compared to that of peers. After observing the peers' successful problem-solving, the learner might believe that he/she can successfully do it as well. Moreover, if he/she observes that he/she could do better than his/her peers, the level of self-efficacy rises (Ozcan, et al., 2021).

Social persuasions involve the positive or negative feedback, encouragements, or appraisals a learner receives from significant people such as peers, teachers, and parents. A learner might ultimately view him/herself through the same lens as that of influential people. The favorable feedback may raise a learner's level of self-efficacy, while unfavorable ones may lower it (Bandura, 1997). The physiological or affective states involve a learner's physical feelings, such as fatigue, stress, anxiety, and anger, that could affect the level of engagement and achievement in problem-solving. A learner might interpret a feeling of stress or anxiety in problem-solving as evidence of incompetence (Ozcan & Kultur, 2021). As such, high and uncontrollable levels of stress or anxiety might adversely affect the ability to think and reason (Bandura, 1977). Hence, the ability to manage the levels of the physiological states while engaging in problem-solving could enable optimum performance.

Considering the possible interaction of the sources of self-efficacy and mathematics achievement, this paper seeks to shed light on sources of self-efficacy and non-routine problem-solving. It considers the suggestions of Ozcan and Kultur (2021) and Plonsky and Oswald (2017), who pointed to inadequate studies that employ multiple regression and structural equation modeling in mathematics self-efficacy and mathematics achievement. In this vein, Plonsky and Oswald (2017) argue that researchers relying heavily on a series of univariate ANOVAs ignore the correlated nature of the relationships of interest. Moreover, the adherence to ANOVA by most researchers might obstruct the potential of our empirical efforts to inform and advance the theory and practice. As such, this research employs both linear and multiple regression analysis. Aiming at advancing the theory, it classifies the learners' mathematics self-efficacy beliefs according to Bandura's (1997) sources of self-efficacy: (1) mastery self-efficacy beliefs (MSE), (2) vicarious self-efficacy beliefs (VSE), (3) social persuasion self-efficacy beliefs (SSE), and (4) physiological state self-efficacy beliefs (PSE). It examines how self-efficacy and each category of self-efficacy relate to non-routine mathematics problem-solving achievement (PSA). It seeks to address the following research questions:

(a) What are the grade 11 learners' levels of non-routine mathematics problem-solving achievement?

(b) What are the grade 11 learners' predominant sources of mathematics self-efficacy beliefs?
(c) What is the relationship between the learners' sources of mathematics self-efficacy and their problem-solving achievement?

Theoretical Background

In everyday life and at workplaces, people encounter novel, unfamiliar problems that call for creativity and critical and higher-order thinking. The ability to solve such kinds of problems is the core aim of teaching and learning mathematics at schools. In preparation for competent analytical people, the South African school curriculum incorporated the teaching, learning, and assessment of non-routine critical thinking skills (Department of Basic Education, 2011). The curriculum upholds mathematics's conceptual and procedural knowledge as the indispensable key to effective problem-solving. The gravity of the mathematical competencies required in solving non-routine problems attracted the attention of many mathematics education researchers (e.g., Shawan et al., 2021; Arifin et al., 2021b; Keles & Yazgan, 2021; Mogari & Chirove, 2017; Abdullah et al., 2017; Elia et al., 2009). Shawan et al. (2021) and Mogari and Chirove (2017) discovered that learners faced difficulties in solving non-routine problems. Moreover, Shawan et al. (2021) identified the high-achieving learners' dominant difficulties in solving non-routine problems as language and information management skills. In particular, the learners have deficiencies in reading, recalling, and relating the facts to problem situations. Arifin et al. (2021b) and Elia et al. (2009) discovered that learners were deficient in problem-solving heuristics. As such, their solutions lacked originality due to the application of formal mathematical procedures, e.g., formulae, symbols, and operations. Pointedly, learners' weaknesses in conceptual and procedural knowledge of mathematics lead to difficulties in solving problems (Keles & Yazgan, 2021; Mogari & Chirove, 2017; Abdullah et al., 2017), resulting in unsatisfactory achievement.

Some papers focussed on establishing the existence of a relationship between self-efficacy (SE) and problem-solving (PS) (Agbornu & Edekor, 2020; Zamnah, 2019; De Brito & De Saiza, 2015), while some quantified the strength of the relationship between them (Ugwuanyi et al., 2020; Amri & Widada, 2019; Susilo & Retnawati, 2018). Sadly, the literature reports mixed findings on the relationship between the two constructs (Bakar et al., 2021; Kaskens et al., 2020; Ozturk et al., 2020; Ozkal, 2019; Ozcan & Gumus, 2019). This paper groups the previous findings into studies that discovered significant or non-significant relationships between the constructs. Some of the studies that discovered a positive significant relationship between self-efficacy and problem-solving achievement were Susilo and Retnawati (2018), Amri and Widada (2019), Guven and Cabakcor (2013), and Nicolaidou and Philippou (2003). These studies reported self-efficacy as a possible predictor of mathematics problem-solving achievement. In essence, raising learners' self-efficacy beliefs could positively influence mathematics problem-solving achievement. Some studies that reported significant relationships between the constructs described the relationship as direct or indirect. For instance, Amri and Widada (2019) and Chen (2003) discovered self-efficacy as having a positive direct effect on mathematics problem-solving achievement. Domenech-Betoret et al. (2017) and Ozcan and Gumus (2019) observed an indirect relationship between them. For instance, Domenech-Betoret et al. (2017) reported that the learners' expectancy-value beliefs (subject value, process expectancy, achievement expectancy, cost expectancy) mediated between self-efficacy and achievement. Hence, teachers should consider both self-efficacy and expectancy-value beliefs in developing a teaching and learning program to enhance learners' mathematics problem-solving achievement.

Ozcan and Gumus (2019) discovered that a learner's metacognitive experience mediated self-efficacy and problem-solving achievement. Thus, teachers cannot overlook the need to promote the development of meta-cognitive skills (e.g., predicting, monitoring, and evaluating the problem-solving process) in conjunction with self-efficacy beliefs to raise learners' mathematics achievement subsequently. Bakar et al. (2021) and Kaskens et al. (2020) report an insignificant relationship between mathematics self-efficacy and problem-solving achievement. In particular,

Bakar et al. (2021) observed metacognitive skills, mathematics attitude, and mathematics beliefs as more significant contributors to mathematics problem-solving achievement than self-efficacy.

Using the structural regression model, Yurt and Sunbul (2014) investigated the direct and indirect effects of mathematics self-efficacy sources on mathematics achievement in combination with other factors such as spatial ability, mathematical problem-solving skill, and reasoning skill. By grouping mathematics achievement, spatial ability, mathematical problem-solving skill, and reasoning skill, they pointed out that self-efficacy sources related directly and positively to each of them. Interestingly, when they regrouped mathematics achievement with mathematical problem-solving skills and spatial ability, they observed that self-efficacy sources had an indirect and positive effect on each of them.

Dagdag et al. (2021) studied the relationship between the prospective teachers' sources of self-efficacy and mathematics problem-solving achievement. Their research participants performed satisfactorily in mathematics. Additionally, they concluded that, among the four sources of self-efficacy, social persuasion was the only significant predictor of non-routine mathematics problem-solving achievement. However, the main difference between their research and the current one is the participants' education level. Ozcan and Kultur (2021) studied 257 grade 12 high school learners' sources of self-efficacy and their achievement in a mathematics test and mathematics course. Using regression analysis, they discovered a significant strong positive linear relationship between mathematics course achievement (dependent variable) and mastery experiences (independent variable). As such, mastery experiences could significantly predict mathematics course achievement. In addition, they also observed that mastery experiences, social persuasions, and physiological state significantly predicted mathematics test achievement. The main points of departure between Ozcan and Kutur's (2021) research are that this focuses on a component of a mathematics course, the non-routine problems, while their focus is on the whole mathematics course. In addition, they used a series of simple linear regression analyses, while this research used multiple regression. Multiple regression considers the complex interrelatedness of the sources of self-efficacy. The fact that the linear regression overlooks.

Briefly, investigating the relationship between self-efficacy and problem-solving is a continuing concern within the mathematics education community due to emerging inconclusive results. Additionally, though several studies on the issue employed varied research approaches, a few of them embraced some more integrated analytic research methods. Thus, this context has great opportunities for future research.

RESEARCH METHOD

This research employs a correlational research design to reveal the relationship between the grade 11 mathematics high achievers' self-efficacy beliefs and problem-solving achievement. A sample of 118 learners from seven rural township secondary schools in Tshwane North district, Gauteng, South Africa, participated in the research. Their school subject teachers identified them as mathematics high achievers based on satisfactory performance on the schools' mathematics end-of-term tests. On average, they selected the top twenty mathematics learners from each school. They all completed the mathematics self-efficacy beliefs questionnaire and then wrote a mathematics problem-solving test. A random sample of two out of the 118 learners participated in semi-structured interviews.

Mathematics high achievers participated in the research to reduce inaccuracy in self-efficacy calibration. The paper implemented Chen's (2003) observation of some inaccuracy among learners in estimating their self-efficacy. Chen (2003) observed that many learners overestimate, while a few underestimate their self-efficacy. Many learners at risk of failure reported unrealistically high self-efficacy, probably because of their over-confidence in mathematics problem-solving achievement (Chen, 2003). Pajares and Gaahan (1999) support this view by assuming that high achievers are more accurate in estimating their self-efficacy than low achievers. They observed that high achievers are better at estimating their mathematics problem-solving capacity than low achievers. As such, that would lessen variations in self-efficacy

calibrations. Inaccurate self-efficacy calibration may adversely affect the research findings. However, many documented studies on mathematics self-efficacy did not consider this factor.

The mathematics self-efficacy beliefs questionnaire (MSEQ) was adapted from Dagdag et al. (2020). It was composed of thirty Likert-type items. Each Learner rated his/her level of agreement with each item from 'strongly agree' to 'strongly disagree.' Using Bandura's (1997) framework of sources of self-efficacy beliefs, the thirty questionnaire items were categorized into four groups, namely mastery experience (5 items), vicarious experience (5 items), social persuasion (10 items), and physiological state (10 items. The MSEQ was valid (α=0.86). The mastery experience, vicarious experience, social persuasion, and physiological state components of MSEQ had Cronbach's alpha of 0.80, 0.65, 0.72, and 0.89, respectively. The Cronbach's alpha that is greater than 0.7 (α>0.7) is acceptable as a good measure of validity (Taber, 2018).

The mathematics problem-solving test was composed of five questions that were of reach to the grade 11 learners. The paper expected learners to model each scenario into mathematical statements and then show step-by-step calculations or logical reasons that lead to the expected solution. It scored each question out of five points using the marking guidelines in Table 1. Three senior mathematics teachers from three different schools validated the problem-solving test. Cronbach's alpha of the test was 0.65. The paper attributes the moderate value of alpha to learners' deficient performance in non-routine problem-solving.

Table 1. Problem-solving test marking guidelines

Indicators	Points
Interprets and translates the task correctly into mathematical statements and/or graphs. Uses mathematical language and reasoning insightfully and/or enhances them with graphics or examples. Shows all necessary steps in calculations. The solution is correct and thorough and/or enhanced by extensions and generalizations. Justifies the solution and considers other workable solutions	5
Interprets and translates the task correctly into mathematical statements and/or graphs. Applies effective and complete strategies and representations. Uses mathematical language and reasoning that signifies content mastery. Omits some necessary steps in calculations. The solution is correct, but the process has some unexplained gaps.	4
Develops and/or displays a partial interpretation and/or translation of the task. Applies partially effective and complete strategies and representations. Uses mathematical language and reasoning that signifies content gaps. Presents an incorrect solution due to minor errors or a correct solution, but some steps are incorrect or contain errors	3
Develops and displays incomplete interpretations and/or translations of the task using inappropriate concepts. Applies incomplete and underdeveloped strategies and representations. Uses incomplete and underdeveloped mathematical language and reasoning. Presents an incorrect or correct answer that is not supported by calculations.	2
Displays no evidence of interpreting and/or translating the task. Applies no identifiable strategy or representation. Uses inappropriate mathematical language and reasoning. Presents an incorrect or correct solution that conflicts with the calculations given	1
Restates the information given in the problem statement. Leaves a blank space.	0

Adapted from Oregon Department of Education (2011)

The research used two interview schedules. The first schedule requested them to justify their ratings on some of the selected MSEQ items. This was done to seek a deep understanding of their espoused self-efficacy beliefs and possibly reveal some salient or hidden beliefs that could impact their problem-solving behavior. The second interview schedule was composed of open-ended questions. It aimed at revealing their sources of motivation and reactions to learning mathematics and solving non-routine problems.

Data Analysis

The paper facilitated the analysis of the MSEQ responses by scoring the positive statements from '5' = 'strongly agree' to '1' = 'strongly disagree.' It scored the negative statements from '1' = 'strongly agree' to '5' = 'strongly disagree.' Additionally, it calculated each learner's mean score on each source of self-efficacy. The source of self-efficacy with the highest mean score was the learner's predominant source of self-efficacy. Furthermore, it measured and analyzed each learner's level of self-efficacy using codes 1 = 'low' and 2 = 'high'. It also calculated each learner's problem-solving mean score and the mean score of all respondents. Their problem-solving achievement levels were similarly categorized using the criterion in Table 2. The problem-solving mean scores were correlated against the self-efficacy mean scores. The simple linear regression used Pearson's product-moment correlation coefficient to measure the strength of the relationship between them. The paper conducted both simple linear and multiple regression analysis techniques to unravel the relationship between the two constructs.

Table 2. Criteria for classifying learners' self-efficacy and problem-solving achievement levels.

Classification	Self-Efficacy/Problem-Solving Score ($\bar{x}$)	Code
Low	$0 < \bar{x} < 3.5$	1
High	$3.5 \leq \bar{x} \leq 5$	2

It recorded and transcribed the responses from the interviews. Furthermore, the main themes that revealed learners' self-efficacy from their responses were noted. It cross-examined the inferred self-efficacy beliefs from the interviews with their espoused self-efficacy beliefs from the self-efficacy questionnaire. Consequently, consistency in responses was regarded as a reliability measure for the interviews. In addition, the findings were triangulated from the MSEQ, retrospective questionnaire, and interviews were conducted to identify converging and/or diverging responses and point to conclusive findings.

RESULTS AND DISCUSSION

Results

The learners held the self-efficacy beliefs questioned in this research at different intensities. The vicarious self-efficacy beliefs held strongly by the learners were "there is a change in my confidence level after watching successful performances in problem-solving (21%)", "I care about others' performance in problem-solving (38%)" and "I care when others can solve problems in mathematics successfully (41%)". The other twenty-nine variables could explain the 21%, 38% and 41% of their variation, respectively. They weakly held the following vicarious self-efficacy beliefs: "The works of others affect my eagerness to solve mathematics problems successfully (63%)", and "Seeing someone successfully solve a problem in mathematics affects my eagerness to do the same (61%)". The other twenty-nine variables could account for their 63% and 61% variation, respectively. Two of them were interviewed and agreed with the variable "The works of others affect my eagerness to solve mathematics problems successfully," equivalently justifying their rating. For example, B42 said:

> "*I rated it 'Agree' because when I see other learners answering a question correctly, it makes me feel that I can solve it well also*".

The social persuasion self-efficacy beliefs which they strongly held were "I believe my teacher when she/he says that the problem has a solution (19%)", "I believe that my success or failure in solving mathematics problems also depends on what other people say (23%)', and "When someone tells me that I can solve a mathematics problem, my confidence is boosted (32%)". The other twenty-nine variables could account for the 17%, 23% and 32% of their variation, respectively. They placed less importance on the variable "The encouraging words I receive from my friends help boost my confidence in problem-solving (56%)". The proportion of the variance explained by the other twenty-nine variables was 29%. When asked to justify their rating on the item: "When someone tells me that I can solve a mathematics problem, my confidence is boosted," they responded as follows:

> B17: "*I rated it 'Agree' because when someone tells me that I can do something that I thought I could not do, I realize that the people around me acknowledge my potential. This motivates me to prove my ability to do it.*"
>
> B42: "*I rated it 'Agree' because, even though it exerts pressure on me, it also motivates me, boosts my confidence, and gives me hope in solving mathematics problems*".

They considered the following physiological state self-efficacy beliefs as of immense importance: "I feel very exhausted in my mathematics class (34%), and "Secondary school algebra problems bring pressure to me (48%)". Their proportions of variance explained by the other twenty-nine variables were 34% and 48%, respectively. They considered these variables as of less importance: "I see mathematics problems as threats (70%)", "I am afraid of mathematics problems (64%)", and "I cannot explain the undesirable feeling I get during problem-solving in mathematics (62%)". The other twenty-nine variables in the MSEQ could account for the 70%, 64% and 62% of their variation, respectively. Asked to justify their rating on "I cannot explain the undesirable feeling I get during problem-solving in mathematics," they responded as follows:

> B17: "*I am uncertain because I usually get an undesirable feeling when I am solving a mathematics problem that I do not understand and cannot figure out a way forward. I get frustrated because of that".*
>
> B42: "*I strongly disagree because when I am solving mathematics problems, I have mixed feelings which are positive and negative.*"

Under the mastery self-efficacy, they expressed great weight on the variable, "If someone asks me to solve a problem in mathematics, I tell myself that I should solve it successfully (30%)'. The other twenty-nine variables could explain a 30% proportion of its variance. They expressed less weight on the variable "I am certain I can solve math problems because I was able to master the skills in problem-solving (70%)". The proportion of variance accounted for by the other twenty-nine variables was 70% (see Table 3). They justified their ratings on 'I am certain I can solve mathematics problems because I was able to master the skills in problem-solving' as follows:

> B17: "*I rated it uncertain because I find it difficult to translate the word statements into algebraic equations*".
>
> B42: "*I am uncertain because I find it a bit difficult to understand and answer the problem correctly in a reasonably expected time*".

Table 3. Extracted commonalities

	Initial	Extraction
I am certain I can solve Mathematics problems because I was able to master the skills in problem-solving.	1.00	0.70
I have mastered solving Mathematics problems that is why I will be able to deal with a problem set successfully.	1.00	0.68
I believe I can solve Mathematics problems successfully because I know I possess the skills.	1.00	0.65
I always tell myself that if I can solve a problem in Mathematics today, then I will be able to solve problems in the future.	1.00	0.61
If someone asks me to solve a problem in Mathematics, I tell myself that I should solve it successfully.	1.00	0.30
Seeing someone successfully solve a problem in Mathematics affects my eagerness to do the same.	1.00	0.61
I care when others can solve problems in Mathematics successfully.	1.00	0.41
The works of others affect my eagerness to solve Mathematics problems successfully.	1.00	0.63
There is a change in my confidence level after watching successful performances in problem-solving.	1.00	0.21
I care about others' performance in problem-solving.	1.00	0.38
Someone who tells me that I can solve a Mathematics problem boosts my confidence.	1.00	0.32
I believe that my success or failure in solving Mathematics problems also depends on what other people say.	1.00	0.23
My teacher increases my confidence by giving me positive feedback about my performance in problem-solving.	1.00	0.40
I feel I can solve Mathematics problems when my friends encourage me.	1.00	0.32
The support I receive from my teacher boosts my confidence in solving Mathematics problems.	1.00	0.45
I believe my teacher when she/he says that the problem has a solution.	1.00	0.19
Being commended for improving in problem-solving enhances the way I see myself.	1.00	0.36
The encouraging words I receive from my friends help boost my confidence in problem-solving.	1.00	0.56
The belief of my friends that I can succeed at problem-solving makes me believe that I possess the necessary skills.	1.00	0.43
Receiving positive feedback about my ability in problem-solving encourages me.	1.00	0.47
I feel threatened when someone gives me a mathematics problem to solve.	1.00	0.50
I get nervous in my Mathematics class.	1.00	0.55
I am afraid of Mathematics problems.	1.00	0.64
I get a racing heart when asked to solve a problem.	1.00	0.59
Secondary School Algebra problems bring pressure on me.	1.00	0.48
I feel very exhausted in my Mathematics class.	1.00	0.34
I get a feeling of discomfort while solving a problem in Mathematics.	1.00	0.59
I see Mathematics problems as threats.	1.00	0.70
I cannot explain the undesirable feeling I get during problem-solving in Mathematics.	1.00	0.62
I cannot concentrate or focus while solving Mathematics problems.	1.00	0.60

Most of them (91.5%) had one predominant source of self-efficacy. The predominant sources of self-efficacy in descending order of frequency are vicarious experience (38.1%), social persuasion (22.0%), physiological state (19.5%), and mastery experience (11.9%). A few of them (8.5%) had at least two predominant sources of self-efficacy (see Table 4).

Table 4. Predominant sources of self-efficacy

		Frequency	Percent	Cumulative Percent
Valid	Mastery experience	14	11.9%	11.9%
	Vicarious experience	45	38.1%	50.0%
	Social persuasion	26	22.0%	72.0%
	Physiological state	23	19.5%	91.5%
	Vicarious & Social persuasion	4	3.4%	94.9%
	Mastery & Vicarious experience	2	1.7%	96.6%
	Mastery, Vicarious & Physiological state	3	2.5%	99.2%
	Mastery, Vicarious, Social persuasion & Physiological state	1	0.8%	100.0%
Total		118	100.0%	

Most (84.7%) had high self-efficacy levels, while 15.3% had low self-efficacy levels (see Table 5). The self-efficacy mean score was 3.95 out of 5, and the standard deviation was 0.44.

Table 5. Mathematics self-efficacy levels

		Frequency	Percent	Cumulative Percent
Valid	Low	18	15.3%	15.3%
	High	100	84.7%	100.0%
Total		118	100.0%	

They showed less competency in formulating algebraic equations from word problems. Question 1 (money problem) had the highest mean score ($\bar{x} = 4.45; \sigma = 1.43$), while question 5 (Tea and cakes problem) had the lowest mean score ($\bar{x} = 1.27; \sigma = 0.98$). Their overall problem-solving mean score was 2.82 out of 5, with a standard deviation of 1.10. Consequently, over two-thirds (71.2%) were classified as having low problem-solving achievement levels (see Table 6).

Table 6. Mathematics problem-solving achievement levels

		Frequency	Percent	Cumulative Percent
Valid	Low	84	71.2%	71.2%
	High	34	28.8%	100.0%
Total		118	100.0%	

There was an insignificant weak positive linear relationship between mastery self-efficacy (MSE) (independent) and problem-solving achievement (PSA) (dependent) ($r = 0.06; p > 0.05$). The MSE could explain a 0.36% change in PSA ($R^2 = 0.0036$). The paper observed an insignificant weak positive linear relationship between social self-efficacy (SSE) and PSA ($r = 0.05; p > 0.05$). The percentage variation explained is 0.25% ($R^2 = 0.0025$). iSimilarly an insignificant weak positive linear relationship between the physiological self-efficacy (PSE) and PSA was observed $r = 0.16; p > 0.05$). The percentage variation in PSA explained by PSE is 2.56% ($R^2 = 0.0256$). In addition, an insignificant weak positive linear relationship wexistedbetween vicarious self-efficacy (VSE) and PSA ($r = 0.05; p > 0.05$). The variation in PSA explained by VSE is 0.25% ($R^2 = 0.0025$). Overall, there was an insignificant weak positive linear relationship between the learners' mathematics self-efficacy (independent) and problem-solving achievement (dependent) ($r = 0.10; p > 0.05$). Self-efficacy explained 1% of the variation in PSA ($R^2 = 0.01$).

Because of the possible interactions between or among the predictors (MSE, VSE, SSE and PSE) (Plousky & Oswald, 2017), an integrated analytical data analysis approach, multiple regression, was employed. Multiple regression allowed the paper to account for the possible correlations between the independent variables and estimate their relative contributions to the variance in the PSA (dependent variable). The multiple correlation coefficient between the independent variables and the dependent variable is 0.22. $R^2 = 0.05$ indicates that the MSE, VSE,

SSE, and PSE predictors explained just 5% of the variance in the PSA. The equation of the regression line is:

$$PSA = 2.13 + 0.03MSE + 0.20VSE - 0.33SSE + 0.28PSE$$

The predictors can be ranked in order of their relative impact on PSA as follows: Physiological Self-efficacy (PSE) ($\beta = 0.28; p = 0.062 > 0.05$), vicarious self-efficacy (VSE) ($\beta = 0.20; p = 0.185 > 0.05$), mastery self-efficacy (MSE) ($\beta = 0.03; p = 0.876 > 0.05$), and social self-efficacy (SSE) ($\beta = -0.33; p = 0.185 > 0.05$) (see Table 7).

Table 7. Problem-solving coefficients

	Unstandardized Coefficients		Standardized coefficients			95% Confidence Interval for B		Collinearity Statistics	
	B	Std Error	Beta	t	Sig.	Lower bound	Upper bound	Tolerance	VIF
(Constant)	2.13	1.03	0.00	2.07	0.041	0.09	4.16		
Mastery experience	0.03	0.20	0.02	0.16	0.876	-0.36	0.42	0.67	1.49
Vicarious experience	0.20	0.18	0.11	1.15	0.254	-0.15	0.56	0.86	1.16
Social persuasion	-0.32	0.24	-0.15	-1.33	0.185	-0.81	0.16	0.71	1.41
Physiological state	0.28	0.15	0.20	1.89	0.062	-0.01	0.57	0.77	1.29

An increase of one unit in PSE, VSE, and MSE will subsequently increase the PSA mean score by 0.28, 0.20, and 0.03 units, respectively. Unfortunately, an increase of SSE by one unit will decrease the PSA mean score by 0.32 units. However, the relative impact of each predictor on PSA was statistically insignificant ($p > 0.05$). Furthermore, $F(4:113) = 1.38; p = 0.245 > 0.05$ indicates that the independent variables are not statistically significant predictors of non-routine mathematics problem-solving achievement. Therefore, MSE, VSE, SSE, and PSE insignificantly predict the PSA of these learners.

Discussion

Most of the learners in this research were of high self-efficacy levels. This finding resonates with Bakar et al. (2021) and Pajares (1996). They held at least one predominant source of self-efficacy. The existence of at least one predominant source of self-efficacy in a learner and possible interactions between or among them could complicate the analysis of the relative contributions of each source of self-efficacy to problem-solving achievement. Though their self-efficacy levels were high, they performed poorly in the non-routine problem-solving test. They faced a great challenge in modeling the given scenarios into algebraic expressions or equations. This adversely affected their performance in problem-solving. This finding is in line with that of Shawan et al. (2021) and Mogari & Chirove (2017) and inconsistent with that of Setyowati et al. (2019). Setyowati et al. (2019) discovered a positive direct relationship between the levels of self-efficacy and mathematics learning outcomes. The conflicting findings could be due to relating self-efficacy to different learning outcomes.

The predictors of problem-solving achievement could be ranked in order of their relative impact as physiological self-efficacy, vicarious self-efficacy, mastery self-efficacy, and social self-efficacy. The paper discovered that MSE, VSE, and PSE positively impact PSA. Hence, increases in levels of these sources of self-efficacy could result in increases in the levels of PSA. Surprisingly, the SSE negatively impacted PSA. Thus, an increase in the levels of SSE could result in a decrease in the levels of PSA. This unexpected finding contradicts that of Dagdag et al, (2021) whose findings suggested that social persuasion is the only significant predictor of non-routine problem-

solving achievement. The magnitudes of the impact of the physiological state and social persuasion sources of self-efficacy are relatively greater than the others. Without overlooking the relative effects of the other sources of self-efficacy, the changes in PSE and SSE might result in drastic changes in learners' PSA. The findings of Ozcan and Kultur (2021), Lai et al. (2020), Amri and Widada (2019), and Yurt and Sunbul (2014) support the existence of a positive relationship between self-efficacy and PSA. Lai et al. (2020) discovered that self-efficacy could predict PSA. Similarly, Amri and Widada (2019) and Yurt and Sunbul (2014) observed a positive direct effect of self-efficacy on PSA. However, unlike the findings of this research, Ozcan and Kultur's (2021) findings indicate that all the sources of self-efficacy are positively related to PSA.

In addition, the findings of this research revealed that all the sources of self-efficacy are not statistically significant predictors of non-routine mathematics problem-solving achievement. Conversely, Ozcan and Kultur's (2021) findings pointed out that MSE, SSE, and PSE are significant predictors of PSA. Only VSE was not significantly related to it. Taken together, the findings of this paper suggest an insignificant weak positive linear relationship between mathematics self-efficacy beliefs and non-routine problem-solving achievement. This finding corresponds to that of Bakar et al. (2021) and Kaskens et al. (2020). According to Kaskens et al. (2020), mathematics self-efficacy could not predict a learner's mathematical problem-solving development. However, it contradicts the findings of Ugwuanyi et al. (2020), Ozkal (2019), Ozturk et al. (2020), Susilo and Retnawati (2018), Guven and Cabakcor (2013), and Pajares (1996) who identified it to be significant. For instance, Guven and Cabakcor (2013) observed a moderately significant relationship between them. Moreover, the findings of Guven and Cabakor (2013) corroborate the historical findings of Pajares (1996), who reported that the self-efficacy of the gifted learners independently contributed to the prediction of the problem-solving in a model that controlled the effects of the other factors such as mathematics anxiety, cognitive ability, self-efficacy for self-regulated learning, and gender. Potential causes of contradictory findings could be, for example, the use of different self-efficacy and problem-solving achievement measuring tools. An insignificant weak positive relationship between self-efficacy and PSA can mean that the predictive power of the sources of self-efficacy to non-routine problem-solving achievement is weak and unreliable.

CONCLUSION

This paper's findings revealed that most high-achieving learners have elevated levels of mathematics self-efficacy and low levels of non-routine problem-solving achievement. Though they achieved high in secondary school mathematics courses, they were incompetent in solving non-routine mathematical problems. The sources of mathematics self-efficacy ranked in order of their relative impact on problem-solving achievement are physiological state, vicarious experiences, mastery, and social persuasion self-efficacy. The physiological, vicarious, and mastery self-efficacy positively influenced problem-solving achievement. Social persuasion self-efficacy negatively impacted problem-solving achievement. The teaching and learning approaches that positively influence the development of physiological, vicarious, and mastery self-efficacy could positively affect the learners' problem-solving achievement. The sources of mathematics self-efficacy and non-routine problem-solving achievement have an insignificant, weak, and positive linear relationship between them. As such, great increases in levels of self-efficacy could result in little or insignificant improvement in problem-solving achievement. The findings of this paper point to an unreliable prediction of non-routine problem-solving achievement using the sources of self-efficacy. The present research confirms some previous findings and extends our knowledge of how self-efficacy relates to problem-solving.

RECOMMENDATIONS

The secondary school mathematics curriculum should accommodate and promote the development of non-routine critical thinking skills in learners. A learner who is well equipped with and trained to use problem-solving tools effectively tends to have confidence in problem-

solving, which cascades to high self-efficacy. Additionally, teachers should expose learners to everyday life problems that require reasoning and the application of heuristics to solve them. Among other skills, they need to develop their ability to model problem situations into mathematical statements. The teachers and learners should extensively promote the positive development of mastery, vicarious, and physiological self-efficacy beliefs to boost problem-solving achievement. Teachers should be cautious of the kind of feedback, judgments, and appraisals they pass to learners because of their possible effect on strengthening or weakening their self-efficacy beliefs. Researchers should investigate further the relative predictive power of each source of mathematics self-efficacy on non-routine mathematics problem-solving achievement using more integrated analytic research methods such as structural equation modeling. The results of this research might apply only to the schools under research.

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